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Excess Sensitivity and Volatility of Long Interest Rates: The Role of Limited Information in Bond Markets

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Abstract

Asymmetric information between the central bank and bond markets creates an inference problem that affects the behaviour of long interest rates. This paper employs a simple macroeconomic model with a time-varying inflation target to illustrate the implications of asymmetry for the sensitivity of long rates and volatility of bond returns. When the central bank's inflation target is not communicated and macroeconomic shocks are imperfectly observed, bond markets infer the value of the target from noisy signals. This heightens the sensitivity of long-run inflation expectations to transitory shocks, thereby raising the measured reaction of long rates to monetary policy and to inflation surprises. Calibrated coefficients from such regressions are more than twice as large when bond markets lack knowledge of the target compared with a full information scenario. Time variation in the inflation target is the main source of volatility, but learning adds to the ability of the model to explain the observed volatility of returns along the yield curve.

Keywords: Term structure of interest rates, yield curve, limited information, learning, excess sensitivity, excess volatility.

JEL classifications: E43, E52.

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1 Introduction

Bond markets exhibit puzzling behaviour in several respects. Long interest rates are more volatile than predicted by standard macroeconomic models paired with the pure expectations hypothesis of the term structure, as is the spread between long and short interest rates (Campbell and Shiller, 1991). In the wake of Shiller's (1979) and Singleton's (1980) findings of excess volatility, an extensive literature has developed to address this behaviour. Some have turned to non-stationarity in short rates or time variation in term and risk premia.[1] Others, including Rudebusch (1995) and Fuhrer (1996), have pointed to time-varying monetary policy goals as a way to reconcile the data with theory.

In addition to being overly volatile, interest rates at long horizons are sensitive to current events – a phenomenon referred to as the excess sensitivity puzzle. Interest rates on bonds as long as 30 years react positively and significantly on average to current monetary policy innovations, as found by Cook and Hahn (1989), Kuttner (2001) and Ellingsen and Söderström (2004). Empirical work by Gürkaynak, Sack, and Swanson (2003) also shows that forward rates up to 15 years ahead respond to today's news about inflation and output and exhibit as much volatility at long horizons as at short.

Against the benchmark predictions of a macroeconomic model with time-invariant parameters and fully informed agents, these behaviours are puzzling. Models incorporating backward-looking behaviour still have difficulty reproducing the lengthy response of long rates.[2] A number of authors, including Romer and Romer (2000), Ellingsen and Söderström (2001) and Gürkaynak, Sack, and Swanson (2003), have pointed to the role that asymmetric central bank information may play in the reaction of long interest rates to monetary policy. Revision of long-run inflation expectations prompted by the revelation of such information seems a likely candidate for explaining the behaviour described above. This view is supported by findings in the empirical finance literature that much of the variation in the term structure is due to changes in expected inflation (see Ang and Bekaert 2004).

The aim of this paper is to address two related questions. First, how is the volatility

[1] Tests of excess volatility are sensitive to the time-series properties of short rates. Variance bounds tests are misleading when short rates are non-stationary (Flavin, 1981, Marsh and Merton 1986, Cushing and Ackert, 1994). In addition, any amount of volatility can be explained with enough term premium variation.

[2] In the partly backward-looking model of Rudebusch (2002), the impulse response of forward rates to shocks dies out completely within 10 years, implying only modest responses of 10-year bonds.

of long interest rates affected by asymmetric information when monetary policy goals are time-varying? Second, do these factors play a role in explaining the excess sensitivity puzzle?

I present a forward-looking dynamic stochastic general equilibrium (DSGE) model with sticky prices and a time-varying and highly persistent inflation target. Two information scenarios are considered, one in which the inflation target is communicated to bond market participants and one in which it is not. The latter generates a signal-extraction problem in which bond markets infer the value of the inflation target from noisy signals. Unlike Ellingsen and Söderström (2001), who consider asymmetric central bank information about preferences and shocks in isolation from one another, this paper generates a true signal-extraction problem in which bond market participants decompose information about inflation into its permanent and transitory elements.

Paired with the expectations hypothesis of the term structure, the model leads to expressions for bond yields which can be used to address volatility and sensitivity. The counterfactual of a communicated, time-varying inflation target is helpful in distinguishing the implications of time-variation from learning about an unobserved target.

The adaptive learning mechanism used in this paper posits that agents adjust their estimates of unknown state variables with a linear updating algorithm that reacts to their forecast errors. In other words, they employ the optimal Kalman filter which minimises the mean squared error of the one-step ahead forecast error of the unobserved target. The signal-extraction problem prevents convergence to the true value of the target but has the implication that agents discount old information. Persistent discrepancies between the true and perceived target and the revision of long-run inflation expectations are key to the main results in the paper.

This is related to some of the the recent literature on learning in macroeconomics. Of relevance to the bond volatility results derived here, Honkapohja and Mitra (2003) show that endogenous variables in an economy exhibit greater volatility when memory is bounded and learning does not converge to the rational expectations equilibrium. Orphanides and Williams (2003) also employ finite-memory, constant-gain learning to show that large transitory shocks result in pronounced swings in inflation expectations despite true parameter stability. This paper is in a similar spirit but extends the analysis to the case where constant-gain learning is warranted by time variation in the unknown state (the inflation target) and draws out the implications for financial market behaviour.

The key findings are as follows. Learning results in heightened sensitivity to transitory shocks, imparting additional variance to forecast errors (Proposition 1) and to the volatility of bond returns for all maturities (Propositions 2 and 3). This is the case even though the rate of learning (the gain in a Kalman filter) is optimally calibrated to the true signal-to-noise ratio in the economy. Time variation in the policy target is the main source of volatility in interest rates but compared with the counterfactual of full information, learning adds to the ability of the model to explain the observed volatility in bond returns.

The sensitive behaviour of forward rates described above can also be replicated by the presence of a non-stationary inflation target and learning. The sensitivity of forward rates to inflation surprises increases substantially when the asymmetric information problem is introduced. Simulated coefficients of the response of long interest rates to surprise innovations in the policy rate are comparable to the estimated values of Kuttner (2001) and Ellingsen and Söderström (2004) at the long end of the yield curve. Moreover, these coefficients are more than twice those that would be observed if participants had full information about policy preferences because constant-gain learning raises the covariance between long and short interest rates (Proposition 4).

The paper is organized as follows. Section 2 introduces the model and solves for its behaviour with optimal policy. Section 3 introduces the information assumptions and analytically illustrates their implications for bond volatility and sensitivity in a simplified version of the model. Section 4 returns to the full version of the model and presents the quantitative implications and Section 5 concludes.

2 A Stylised Macroeconomy

In this section I present a stylised model of the macroeconomy and solve for its behaviour with optimal policy in terms of the shocks arriving in the model. The model consists of a forward-looking DSGE model based on agents' optimising behaviour, akin to that of Clarida, Gali, and Gertler (1999) but with the economic environment modified to include a time-varying inflation target. Whilst it is more forward-looking than other commonly simulated models of the macroeconomy (Rudebusch, 2002, for example, is a popular choice), this characteristic keeps the model analytically tractable and key features transparent.[3] Estrella and Fuhrer (1999) criticise the ability of forward-looking new Keynesian models to match the persistence of inflation but this criticism is less potent once persistence is added to policy goals.

2.1 Economic Environment

The model is summarised by the following equations, both of which have their roots in the microeconomic foundations of dynamic general equilibrium theory:

$$x_t = -\gamma[i_t - E_t(\pi_{t+1})] + E_t(x_{t+1}) + g_t, \tag{1a}$$

$$\pi_t - \pi_t^* = \beta E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) + \lambda x_t + u_t, \tag{1b}$$

where π_t is inflation, π_t^* the time-varying inflation target, x_t the output gap (defined as the log deviation of output around potential) and i_t the policy controlled nominal short interest rate. Aggregate demand (1a) is derived from the log-linearised consumption Euler equation that solves the consumption-saving decision of the representative household. The pricing equation (1b) is the log-linear approximation of the aggregate firm pricing rule that arises from individual firms' optimal pricing decisions given staggered price setting similar to Calvo (1983). Appendix A sets out the maximisation problems that lead to both (1a) and (1b). The economic environment in which firms optimise has been augmented to include a time-varying inflation target as in Smets and Wouters (2003) and Adolfson, Laséen, Lindé, and Villani (2004). This differs importantly from the economy described by Clarida, Gali and Gertler (1999), in which the inflation target is assumed to be constant. In brief, firms who are unable to re-optimise prices in a given period instead index their prices to a combination of past inflation and the inflation target. When prices are indexed fully to the current inflation target, the forward-looking nature of the Phillips curve and the tractability of the model are preserved.[4]

[3] Hybrid variants of such models which include both forward- and backward-looking elements can generally not be solved analytically but must rely on numerical methods (see Söderlind (1999)).

[4] If non-optimised prices are partially indexed to past inflation this results in a familiar, partially backward-looking Phillips curve. The assumption that price-setters know the value of the inflation target is strong

The disturbance terms obey the following laws of motion:

$$u_t = \rho u_{t-1} + \hat{u}_t \text{ where } \hat{u}_t \sim iid(0, \sigma^2_{\hat{u}}) \text{ and } \rho \in [0,1) \tag{2}$$

$$g_t = \mu g_{t-1} + \hat{g}_t \text{ where } \hat{g}_t \sim iid(0, \sigma^2_{\hat{g}}) \text{ and } \mu \in [0,1) \tag{3}$$

where $\hat{u}_t$, $\hat{g}_t$ are independent (i.e.: $E[v_t v_t'] = \begin{bmatrix} \sigma^2_{\hat{u}} & 0 \\ 0 & \sigma^2_{\hat{g}} \end{bmatrix}$ where $v_t' = [\hat{u}_t \ \hat{g}_t]$).

The highly persistent and possibly non-stationary nature of inflation in many industrialized countries suggests that a model with a constant steady state and mean-reverting nominal short rates may be the wrong benchmark.[5] Here the inflation target is modeled as a random walk with innovation variance σ^2_ε:

$$\pi^*_t = \pi^*_{t-1} + \varepsilon_t \quad where \;\; \varepsilon_t \sim iid(0, \sigma^2_\varepsilon). \tag{4}$$

Several sources provide evidence for non-stationarity of the inflation target. Smets and Wouters (2003) find that a large share of the movement in inflation in both the U.S. and Euro-area economies over the last 20 years can be explained by permanent shifts in a non-stationary process for the inflation target. Kozicki and Tinsley (2003) have similar success in describing the evolution of long run inflation expectations in the U.S. by modelling the unknown inflation target as a random walk. Fuhrer (1996) also finds that the implied series for the inflation target contains a unit root in an exercise to back out counterfactual policy parameters that reconcile the pure expectations hypothesis with movements in U.S. long interest rates.

The specification in equation (4) has the advantage that it does not presume knowledge of the number or type of potential policy regimes, as is the case when the target is modeled as a Markov switching process. A shortcoming is that the process is unbounded at long horizons. From this point of view, it may be more attractive to model the inflation target as a persistent, mean-reverting process. However, a highly persistent auto-regressive process

given the later scenario in which bond market participants must infer the target. However, when price-setters' inflation expectations are also formed through adaptive learning, as for example in Preston (2002), the evolution of the macroeconomy becomes substantially more complicated and detracts from the exposition of later results.

[5] For persistence see Fuhrer and Moore (1995) and for evidence of unit roots see Mishkin (1992) and Wallace and Warner (1993).

exhibits such slow mean-reversion that in finite samples its behaviour is indistinguishable from a unit root process. Rudesbusch and Wu (2003), for example, permit such a specification and find a persistence parameter of 0.989.[6]

For sufficiently small values of σ_ε^2 the target tends to stay within plausible bounds (the empirical evidence discussed in Section 4 suggests a standard deviation of around ±1 per cent per decade). It also captures the idea that an apparently stable monetary policy regime may continue to exhibit small, permanent adjustments in its preferences and inflation target, as suggested by Cogley, Morozov and Sargent (2003).

Permanent shocks to the inflation target are the only type of policy change in the model, although to a first approximation changes in the relative preference for output stability in the loss function could be thought of as adjustments in the level of the target. All other structural parameters in the economy are assumed constant and known.

The timing of the model is that all shocks ($\hat{u}_t$, $\hat{g}_t$ and ε_t) are realised at the beginning of period t; taking these into account, the central bank sets its policy interest rate to affect the outcomes $\{\pi_t, x_t\}$ in the same period. If it is communicated, the inflation target is announced at the beginning of the period.

2.2 Optimal Policy with Discretion

Consider the standard optimal policy problem of a central bank aiming to minimise the following loss function with discretion

$$L = -\frac{1}{2} E_t \sum_j \Psi^j \left(\alpha x_{t+j}^2 + (\pi_{t+j} - \pi_{t+j}^*)^2 \right) \tag{5}$$

where α is the relative preference weight on output stability. The central bank chooses the pair $\{x_t, \pi_t\}$ each period to minimise its loss function and sets the appropriate value of the policy controlled interest rate i_t to achieve this. Given the purely forward-looking nature of the economy, monetary policy has only a contemporaneous effect and the intertemporal

[6] Other authors have suggested augmenting the specification to include adjustment to cost-push shocks as in Kozicki and Tinsley (2003) or lagged inflation as in Gürkaynak, Sack, and Swanson (2003). For clarity of exposition, the process considered here is a simple random walk. An alternative way of modelling inflation, and perhaps the target, is by combining mean-reverting dynamics with infrequent intercept shifts as in Levin and Piger (2002). This specification is still fundamentally non-stationary.

policy optimisation problem reduces to a sequence of static optimisations. This leads to a first order condition representing the standard policy trade-off between the output gap and inflation gap:

$$x_t = -\frac{\lambda}{\alpha}(\pi_t - \pi_t^*). \tag{6a}$$

The model can be solved for y_t and π_t in terms of current shocks (see Appendix B) to yield

$$\pi_t - \pi_t^* = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} u_t \tag{7}$$

$$x_t = -\frac{\lambda}{\alpha}[\pi_t - \pi_t^*] = \frac{-\lambda}{\lambda^2 + \alpha(1-\beta\rho)} u_t. \tag{8}$$

Clarida, Gali, and Gertler (1999) describe this policy as one of "leaning against the wind," with the central bank choosing how much of the inflation shock to offset in any period according to its preferences and the parameters of the economy. Larger values of α imply a larger inflation gap for a given shock u_t. Higher serial correlation in inflation shocks also raises the multiplier in (7).

The optimal monetary policy reaction function takes a familiar form for this class of model, resembling a Taylor rule in the sense that the policy controlled interest rate responds to the current inflation and output gaps. Here the nominal instrument is also pegged at the level of the current inflation target (the constant real interest rate is subsumed in the linearisation):

$$i_t = E_t\left(\pi_{t+1}^*\right) + \left[1 + \frac{\lambda(1-\rho)}{\alpha\gamma\rho}\right] E_t\left[\pi_{t+1} - \pi_{t+1}^*\right] + \frac{g_t}{\gamma}.$$

Note that the coefficient on expected inflation exceeds unity for positive values of λ, α, γ and ρ, a necessary condition for a stabilising rule. Rewriting this in terms of shocks in the model,

$$i_t = \pi_t^* + \left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right] \delta u_t + \frac{g_t}{\gamma} \tag{9}$$

where g_t, can be interpreted as either the aggregate demand shock or a policy control error.

3 Inference, Volatility and Sensitivity

3.1 Information Assumptions

The key relationships of the economy in equations (7), (8) and (9) can be summarised as

$$\begin{aligned}\pi_t &= \pi_t^* + \delta u_t \quad \text{where } \delta = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} \\ \pi_t^* &= \pi_{t-1}^* + \varepsilon_t \\ \text{and} \quad i_t &= \pi_t^* + \left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right]\delta u_t + \frac{g_t}{\gamma}\end{aligned}$$

where u_t, g_t and ε_t have the properties described in Section 2.1. In this environment, forecasting the nominal short rate is a matter of forecasting the time-varying inflation target and macroeconomic shocks in the economy.

The information assumptions are as follows. The central bank is assumed to know the structure of the economy at time t and can observe all current variables and shocks but has no advantage over bond market analysts in forecasting them. That is, the central bank possesses potentially superior information about the current state of the economy which may result in more accurate forecasts.[7] Bond market participants are assumed to be unable to observe shocks but I consider two different scenarios regarding their information about the inflation target:

i) Full information: the inflation target is communicated by the central bank every period and thus is fully observable to bond market participants. Shocks, whilst not observable, can be perfectly inferred from a combination of inflation, the inflation target and the nominal policy short rate.

ii) Limited Information: the inflation target is not communicated. Thus bond market participants are unable to accurately decompose observed inflation into its permanent policy and transitory shock components. In all other respects bond market actors are homogeneously well informed, knowing the structure and parameterization of the economy as well as the central bank's preference for output stability. Furthermore, they

[7] This is consistent with the evidence provided by Romer and Romer (2000) of the superiority of Federal Reserve information due to better data processing.

believe correctly that the inflation target follows a random walk and know the relative variance of innovations to the target and aggregate supply shocks, $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}$.

The first scenario is clearly information rich, more so than would be expected if a central bank were conducting policy with a time-varying inflation target. However, it will serve as a useful counterfactual benchmark. This scenario also nests the case of a central bank conducting explicit constant-inflation targeting.

The second scenario is a more realistic depiction of policy communication for most central banks and requires that bond markets learn about shifts in policy preferences over time in order to form long-run inflation expectations. When the inflation target is time-varying and non-stationary it becomes optimal to place greater weight on recent observations and discount older observations.[8] The model could be generalised to permit some commonly observed shocks without detracting from the effects of asymmetric information but the focus here will be on the case described above.

3.2 Forecasting Inflation and Nominal Short Interest Rates

This section illustrates how bond market participants form inflation expectations and forecast the nominal short interest rate. Within each scenario, all participants have access to the same information and respond homogenously. In Section 3.3, a special case of the general problem is presented which leads to analytical expressions for forecast errors and zero coupon bond yields.

Full Information: Forecasting is straightforward for the case when bond markets know the inflation target at time t. Denote the current information set as Ω_t^{FI}, which includes all information up to and including period t. Given the random walk property of the inflation target, the optimal forecast j-periods ahead conditional on Ω_t^{FI},denoted $\pi_{t+j/t}^{*FI}$, is

$$\begin{aligned}\pi_{t+j/t}^{*FI} &= \pi_t^* + E_t\left(\sum_{i=1}^{j}\varepsilon_{t+i} \mid \Omega_t^{FI}\right) \\ &= \pi_t^* \qquad \text{for all } j \geqslant 1. \end{aligned} \tag{10}$$

[8]Such a strategy could also be motivated by arguing that agents suffer from finite memory, as in Orphanides and Williams (2003), or suspect structural change. Generally, however, the optimal learning strategy when there is no structural change is to allow the sample to grow indefinitely with equal weights on all observations.

The optimal projection of inflation conditional on Ω_t^{FI}, $\pi_{t+j/t}^{FI}$, is found by leading (7) and employing the serial correlation of u_{t+j}:

$$\begin{aligned}\pi_{t+j/t}^{FI} &= \pi_{t+j/t}^{*FI} + \delta u_{t+j/t} \\ &= \pi_t^* + \delta\rho^j u_t \qquad \text{for all } j \geqslant 1.\end{aligned} \tag{11}$$

Similarly leading the policy reaction function in (9) and substituting the optimal projections $\pi_{t+j/t}^{*FI}$, $\pi_{t+j/t}^{FI}$, $u_{t+j/t}$ and $g_{t+j/t}$ yields

$$i_{t+j/t}^{FI} = \pi_t^* + \left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right]\delta\rho^j u_t + \frac{1}{\gamma}\mu^j g_t \quad \text{for all } j \geqslant 1. \tag{12}$$

The effect of transitory inflation and output shocks on the predicted path of short rates dies out geometrically. If shocks are serially uncorrelated (i.e., $\rho = \mu = 0$) the forward rate for horizons $j \geqslant 1$ is simply today's inflation target. A positive, permanent shock to the inflation target raises the expected nominal short rate at all horizons.

Limited Information: To forecast the nominal short rate in the limited information scenario, bond markets must estimate the inflation target and unobserved shocks. They do so recursively, employing a linear algorithm to update their estimate of the unobserved state variables via their forecast errors. This is a straightforward application of a Kalman filter to the specific state space of this model (see Hamilton 1994 Chapter 13 for a thorough discussion).

The stylised economy described above can be given a state space representation in which inflation and the nominal short rate are observable variables whilst the inflation target, price shocks and demand shocks are unobservable state variables.[9]

The observation equations are

$$\begin{bmatrix} \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1 & \delta & 0 \\ 1 & \left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right]\delta & \frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \pi_t^* \\ u_t \\ g_t \end{bmatrix} \tag{13}$$

[9]The output gap (x_t) is assumed to be unobservable in this setup because of the difficulty in observing potential output and agents do not employ it as part of their filtering program to extract π_t^*. However, x_t could be decomposed into measured output and unobservable potential output with the appropriate filter, much as is done today by professional economists.

and the state equations are

$$\begin{bmatrix} \pi^*_{t+1} \\ u_{t+1} \\ g_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \pi^*_t \\ u_t \\ g_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \hat{u}_{t+1} \\ \hat{g}_{t+1} \end{bmatrix} \tag{14}$$

where ε_{t+1}, $\hat{u}_{t+1}$ and $\hat{g}_{t+1}$ are distributed *iid* as before.

The optimal Kalman updating algorithm is

$$\begin{bmatrix} \pi^*_{t+1/t+1} \\ u_{t+1/t+1} \\ g_{t+1/t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \pi^*_{t/t} \\ u_{t/t} \\ g_{t/t} \end{bmatrix} + \begin{bmatrix} \kappa_{\pi,\pi} & \kappa_{\pi,i} \\ \kappa_{u,\pi} & \kappa_{u,i} \\ \kappa_{g,\pi} & \kappa_{g,i} \end{bmatrix} \begin{bmatrix} \pi_t - \pi^{LI}_{t/t-1} \\ i_t - i^{LI}_{t/t-1} \end{bmatrix} \tag{15}$$

where $\begin{bmatrix} \kappa_{f,\pi} & \kappa_{f,i} \end{bmatrix}$, $f = \pi, u, g$ are the steady state Kalman gains.[10] The notation $z_{t/t}$ is used to denoted the inferred value of the state variable at t based on information up to and including t. Individuals update their estimate of the inflation target by attributing a constant fraction of the forecast error in inflation and nominal short rates to the state variables. The following section illustrates how the gain is related to fundamental variances in the model.

This system nests three related learning strategies. When the first equation in (13) is treated as the only observation equation, agents learn about unobserved state variables through the signal contained in inflation and its forecast errors. When the second equation is treated in isolation, the estimate of the inflation target and shocks in the economy are updated through unexpected innovations to the nominal short rate. This corresponds to the ideas of Romer and Romer (2000) and Ellingsen and Söderström (2001), where actions of the central bank contain a signal about their superior information. In the simplified environment described by Ellingsen and Söderström (2001), *either* policy preferences *or* a macroeconomic shock are unobservable but not both at the same time. Thus the signal extraction problem is trivial as the missing information is revealed immediately and completely upon observation of the central bank's policy movement.[11]

[10] For the Kalman filter to be the minimum variance estimator of the unknown states, the errors must be distributed normally. For all other distributions it is the best *linear* estimator. To possess steady state values, the eigenvalues of the coefficient matrix in (14) must be in or on the unit circle.

[11] Technically, the signal-to-noise ratio is either zero or infinity, implying a gain of zero or one respectively.

Gürkaynak et al (2003) and Kozicki and Tinsley (2003) also assume learning via the short rate because in their more backward-looking models it is a more timely signal of policy change. However, in this model, the policy short rate is a noisier signal of the inflation target due to the additional variation contributed by g_t. The third learning strategy is to employ both observation equations in tandem. Bond markets almost surely elicit information from both sources since they react to both inflation news and monetary policy innovations but the timing of policy announcements rarely coincides with inflation releases as in this model.

Forecasting is straightforward and the optimal projections take a similar form to the full information case. From the Law of Iterated Expectations we have that

$$E_t \begin{bmatrix} \pi^*_{t+j} \\ u_{t+j} \\ g_{t+j} \end{bmatrix} \mid \Omega_t^{LI} = \begin{bmatrix} \pi^*_{t+j/t} \\ u_{t+j/t} \\ g_{t+j/t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \mu \end{bmatrix}^j \begin{bmatrix} \pi^*_{t/t} \\ u_{t/t} \\ g_{t/t} \end{bmatrix}. \tag{16}$$

The optimal projection of inflation is thus

$$\begin{aligned} \pi^{LI}_{t+j/t} &= \pi^{*LI}_{t+j/t} + \delta u^{LI}_{t+j/t} \\ &= \pi^*_{t/t} + \delta \rho^j u_{t/t} \qquad \text{for all } j \geqslant 1. \end{aligned} \tag{17}$$

The predicted path of short rates resembles that for full information but state variables are replaced by their inferred values:

$$i^{LI}_{t+j/t} = \pi^*_{t/t} + \left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right] \delta\rho^j u_{t/t} + \frac{1}{\gamma}\mu^j g_{t/t} \quad \text{for all } j \geqslant 1. \tag{18}$$

In the following section we will turn to a simple case to illustrate analytically the connection between learning, excess sensitivity and volatility. In the empirical section we return to the full version of the model and find that the different learning strategies imply very similar results.

3.3 Basic case

In this section the system is simplified in two ways, by taking the case when bond markets update only through inflation forecast errors and assuming that disturbances are serially uncorrelated (i.e., $\rho = \mu = 0$). With these assumptions, the state space simplifies to one observation and one state equation:

$$\text{Observation equation:} \qquad \pi_t = \pi_t^* + \delta \hat{u}_t \tag{19}$$

$$\text{State equation:} \qquad \pi_t^* = \pi_{t-1}^* + \varepsilon_t \tag{20}$$

where $\hat{u}_t$ is interpreted as the serially uncorrelated disturbance of the observation equation. The optimal linear projections of inflation, the inflation target and nominal forward rates simplify in the full information scenario to

$$\begin{aligned} \pi_{t+j/t}^{FI} &= \pi_{t+j/t}^* = \pi_t^* \quad \text{for all } j \geqslant 1 \\ i_{t+j/t}^{FI} &= \pi_t^* \quad \text{for all } j \geqslant 1 \end{aligned} \tag{21}$$

and when the target is not observed,

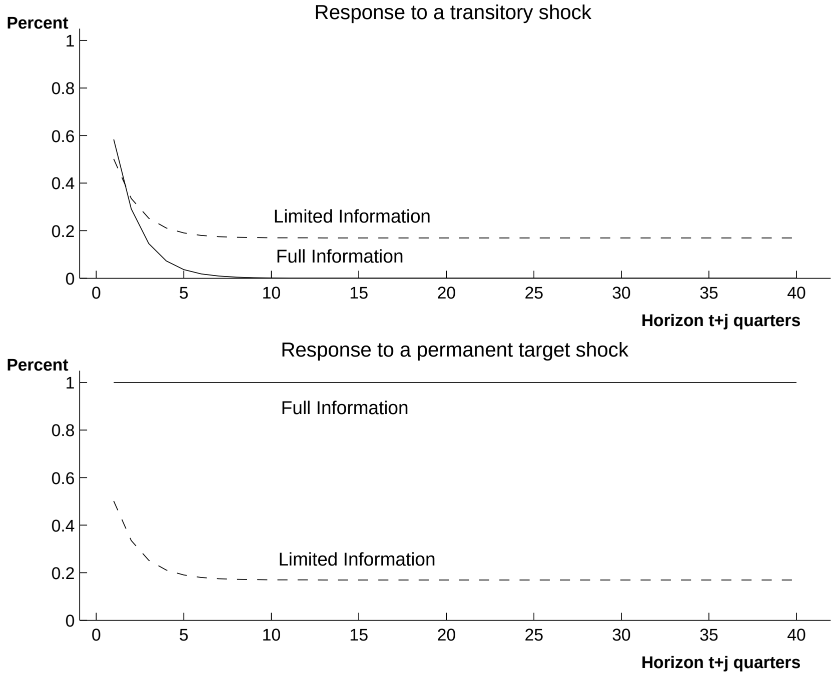
$$\begin{aligned} \pi_{t+j/t}^{LI} &= \pi_{t+j/t}^{*LI} = \pi_{t/t}^* \quad \text{for all } j \geqslant 1 \\ i_{t+j/t}^{LI} &= \pi_{t/t}^* \quad \text{for all } j \geqslant 1. \end{aligned} \tag{22}$$

Bond market participants aim to decompose the observation equation (19) into transitory shocks to inflation and permanent shifts in the inflation target. In this univariate state-space, the optimal Kalman filter algorithm for updating the estimate of the unknown inflation target takes a simpler form:

$$\pi_{t/t}^* = \pi_{t-1/t-1}^* + (1-\phi)(\pi_t - \pi_{t/t-1}^{LI}) \tag{23}$$

where $(1-\phi)$ is the steady state Kalman gain that regulates the proportion of the inflation forecast error attributed to the inflation target. The optimally calibrated gain is a non-linear function of the signal-to-noise ratio, $\phi = \phi\left(\frac{\sigma_\varepsilon^2}{\delta^2 \sigma_u^2}\right)$, bounded between 0 and 1 (see Appendix C for functional form). As innovations to the inflation target become noisier relative to aggregate

Figure 1: Impulse response functions of forward rates



Notes: Response to a one unit shock to u_t and ε_t respectively. Calibrated to the parameters shown in Table 1 with $\sigma_\varepsilon^2 = 0.2$.

supply shocks, more forecast error is attributed to change in the inflation target; that is, as $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2} \rightarrow \infty$, $(1-\phi) \rightarrow 1$ and $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2} \rightarrow 0$, $(1-\phi) \rightarrow 0$.

With constant gain learning as in (23), the perceived target overreacts to transitory shocks (u_t) and underreacts to true changes in the target (ε_t). This is depicted in Figure 1 which shows the impulse responses of forward rates to transitory and permanent shocks for a more general case in which $\rho = \mu > 0$. Over time, the perceived inflation target, $\pi^*_{t/t}$, roughly tracks the actual target but with deviations that reflect current and past inference inaccuracies.

The Kalman filter minimises the variance of the one-step-ahead forecast error of the state variable, here the inflation target. As a result, the unconditional variance of the change in the perceived target matches the second moment of true target innovations. The recursive nature of updating also means that the current estimate of the target can be expressed as a geometric lagged polynomial of the history of observed inflation outcomes:

$$\pi^*_{t/t} = \frac{(1-\phi)}{1-\phi L}\pi_t. \tag{24}$$

3.3.1 Forecast Errors

One-period-ahead inflation forecast errors are central to updating the perceived target and the nominal short rate forecast error is a linear combination of the same components. Comparing its variance in the full and limited information scenarios gives us a sense of how learning adds noise through revision of the perceived target.

Full Information: The one-period-ahead forecast of inflation can be written in terms of the shocks arriving at $t+1$:

$$\pi_{t+1} - \pi^{FI}_{t+1/t} = \left(\pi^*_{t+1} + \delta\hat{u}_{t+1}\right) - \pi^*_t = \varepsilon_{t+1} + \delta\hat{u}_{t+1}.$$

Similarly, the difference between the policy rate dictated by the reaction function in (12) and the forecast of the nominal short rate in (21) yields a forecast error which also reflects shocks to aggregate demand:

$$i_{t+1} - i^{FI}_{t+1/t} = \varepsilon_{t+1} + \frac{\lambda}{\alpha\gamma}\delta\hat{u}_{t+1} + \frac{1}{\gamma}\hat{g}_{t+1}. \tag{25}$$

This forecast error has variance

$$var(i_{t+1} - i_{t+1/t}) = \sigma^2_\varepsilon + \left(\frac{\lambda}{\alpha\gamma}\right)^2 \delta^2\sigma^2_{\hat{u}} + \frac{\sigma^2_{\hat{g}}}{\gamma^2}$$

which is increasing in all innovation variances. More variance in the target is associated with larger forecast errors. The structural and preference parameters that enter the reaction function scale $\sigma^2_{\hat{u}}$ and $\sigma^2_{\hat{g}}$. A stronger preference for output stability (higher α) reduces the variance of the forecast error – higher α calls for proportionately less movement in the nominal short rate via $\frac{\lambda}{\alpha\gamma}$ for a given shock and is less than fully offset by the widening of the tolerated inflation gap $\left(\delta = \frac{\alpha}{\lambda^2+\alpha}\right)$.

Limited Information: The forecast error can be given similar expression in the limited information scenario, reflecting innovations at $t+1$ plus a new term that measures the gap between the true and inferred values of the inflation target $\left(\pi^*_t - \pi^*_{t/t}\right)$:

$$\pi_{t+1} - \pi^{LI}_{t+1/t} = \varepsilon_{t+1} + \left(\pi^*_t - \pi^*_{t/t}\right) + \delta\hat{u}_{t+1}$$

$$i_{t+1} - i^{LI}_{t+1/t} = \varepsilon_{t+1} + \left(\pi^*_t - \pi^*_{t/t}\right) + \frac{\lambda}{\alpha\gamma}\delta\hat{u}_{t+1} + \frac{1}{\gamma}\hat{g}_{t+1}. \tag{26}$$

The lagged polynomial form of $\pi^*_{t/t}$ implies that $\left(\pi^*_t - \pi^*_{t/t}\right)$ is a function of the history of shocks to the economy and its presence adds noise to forecast errors. The variance can be derived using recursive substitution (see Appendix D):

$$var(i_{t+1} - i^{LI}_{t+1/t}) = \sigma^2_\varepsilon(\frac{1}{1-\phi^2}) + \delta^2\sigma^2_{\hat{u}}\left[\frac{\lambda}{\alpha\gamma}^2 + \frac{(1-\phi)^2}{1-\phi^2}\right] + \frac{\sigma^2_{\hat{g}}}{\gamma^2}. \tag{27}$$

Proposition 1 *The variance of the nominal short rate forecast error is unambiguously larger when bond markets learn about an unobserved inflation target than when the target is perfectly observed. That is, $var(i_{t+1} - i^{LI}_{t+1/t}) > var(i_{t+1} - i^{FI}_{t+1/t})$. Specifically,*

$$\sigma^2_\varepsilon(\frac{1}{1-\phi^2}) + \delta^2\sigma^2_{\hat{u}}\left[\frac{\lambda}{\alpha\gamma}^2 + \frac{(1-\phi)^2}{1-\phi^2}\right] + \frac{\sigma^2_{\hat{g}}}{\gamma^2} > \sigma^2_\varepsilon + \frac{\lambda}{\alpha\gamma}^2\delta^2\sigma^2_{\hat{u}} + \frac{\sigma^2_{\hat{g}}}{\gamma^2}$$

for positive σ^2_ε, $\sigma^2_{\hat{u}}$ and $\sigma^2_{\hat{g}}$ and constant gain $\phi \in (0,1)$. Proof in Appendix D.

Any degree of learning generates additional forecast error variance by enlarging the coefficients attached to σ^2_ε and $\sigma^2_{\hat{u}}$. The source of this additional variance is $\left(\pi^*_t - \pi^*_{t/t}\right)$. Intuitively the gain, $(1-\phi)$, determines the extent to which the historical sequence of transitory shocks u_{t-j}, $j = 0, ..., \infty$ are attributed to $\pi^*_{t/t}$. A stronger signal-to-noise ratio (a rise in σ^2_ε for a given value of $\delta^2\sigma^2_{\hat{u}}$) lowers $\left(\frac{1}{1-\phi^2}\right)$ but not sufficiently to offset the rise in σ^2_ε.

As a constant inflation target is approached (i.e., $\sigma^2_\varepsilon \to 0$ and $\phi\left(\frac{\sigma^2_\varepsilon}{\delta^2\sigma^2_u}\right) \to 1$), the forecast error variances of the two information scenarios coincide. When long-run inflation expectations are well-anchored by a constant target, forecast errors reflect only the arrival of transitory shocks.

3.3.2 Bond Yields and the Volatility of Returns

Assuming that the expectations hypothesis of the term structure holds, it is straightforward to characterise the interest rates on bonds and derive the volatility of returns for the different information scenarios. Denoting the interest rate on a zero-coupon bond with maturity m at time t as i^m_t, this interest rate is set as the average expected future short interest rate during the time to maturity plus a term premium:

$$i_t^m = \frac{1}{m}\sum_{j=0}^{m-1} i_{t+j/t} + \zeta_t^m$$

where $i_{t+j/t}$ is, as before, the expected short interest rate j periods ahead and ζ_t^m is the term premium at time t for maturity m. I do not attempt to model time-variation in the term premium but assume that it is independent of all relevant variables in the model. In the calibrations below, time variation in the inflation target rather than the term premium is used to match historical volatility data, although time variation in the latter would not change the spirit of the exercise.

To build long rates, bond market participants form expectations about the future path of short interest rates based on their current information set, Ω^{FI} or Ω^{LI}. With the real interest rate assumed constant, forecasts of shocks and inflation expectations drive movements in the term structure.[12]

Full Information: Denote an m-period bond in the full information scenario as $i_t^{m^{FI}}$. Combining the short rate at period t from the central bank reaction function in (9) with the optimal projection of nominal short rates in (21) yields the following:

$$\begin{aligned} i_t^{m^{FI}} &= \frac{1}{m}\left(i_t + i_{t+1/t}^{FI} + \ldots + i_{t+m-1/t}^{FI}\right) + \zeta_t^m \\ &= \pi_t^* + \frac{1}{m}\left(\frac{\lambda}{\alpha\gamma}\delta\hat{u}_t + \frac{\hat{g}_t}{\gamma}\right) + \zeta_t^m. \end{aligned} \tag{28}$$

The nominal yield is pegged at the level of the current inflation target and reflects transitory shocks, $\hat{u}_t$ and $\hat{g}_t$, only to the extent that they drive the current nominal short rate away from its equilibrium level. As m increases, averaging ensures that the effect of these shocks on longer yields diminishes.

Non-stationarity of the nominal short rate implies that variances in levels are unbounded as t goes to infinity. Instead, the focus here will be on the variance of the period-to-period change in an m-period bond, a measure of the volatility of returns.[13] Differencing (28) yields

[12]This is broadly consistent with the findings of Ang and Bekaert (2004) who detect very little movement in the real component of the term structure. They also conclude that the majority of the variance in long term nominal interest rates is due to changes in expected inflation rather than inflation risk premia.

[13]Denoting the price of the zero coupon bond B_t^m we have $B_t^m = \exp(-i_t^m m)$ where i_t^m is the yield on the

an expression for the change in an m-period bond between two periods:

$$i_t^{mFI} - i_{t-1}^{mFI} = \varepsilon_t + \frac{1}{m}\left(\frac{\lambda}{\alpha\gamma}\delta\left(\hat{u}_t - \hat{u}_{t-1}\right) + \frac{\hat{g}_t - \hat{g}_{t-1}}{\gamma}\right) + \zeta_t^m - \zeta_{t-1}^m. \quad (29)$$

To features are worth pointing out. The bond yield at any maturity changes one-for-one with the inflation target while the effect of transitory shocks diminish with maturity:

$$\frac{\partial\left(i_t^{mFI} - i_{t-1}^{mFI}\right)}{\partial\varepsilon_t} = 1$$
$$\frac{\partial\left(i_t^{mFI} - i_{t-1}^{mFI}\right)}{\partial\hat{u}_t} = \frac{1}{m}\frac{\lambda}{\alpha\gamma}\delta.$$

From here the variance of the change in bond yield will be referred to as the volatility of its return. From the expression for the change in the bond yield we have the following proposition.

Proposition 2 *The volatility of returns to an m-period bond with full information and serially uncorrelated errors is:*

$$var(i_t^{mFI} - i_{t-1}^{mFI}) = \sigma_\varepsilon^2 + \frac{1}{m^2}\left(2\left(\frac{\lambda}{\alpha\gamma}\right)^2\delta^2\sigma_{\hat{u}}^2 + \frac{1}{\gamma^2}2\sigma_{\hat{g}}^2\right) + \sigma_\varsigma^2.$$

where σ_ς^2 *is the variance of the term premium. Proof follows immediately from equation (29) and the independence of shocks to the economy.*

First, note that this variance is rising one-for-one with σ_ε^2 which could be interpreted as a substitute for σ_ς^2. As σ_ε^2 approaches zero, bond volatility declines until the only source of variation is the realisation of unforecastable transitory shocks and term premia variation. This has the natural implication that for given variances of transitory shocks, bond volatility under inflation targeting should be less than when the inflation target is allowed to vary over time.

Secondly, volatility is rising in $\sigma_{\hat{u}}^2$ and $\sigma_{\hat{g}}^2$, both of which are scaled by their respective effect on the current short rate. The effect of transitory shocks on bond volatility diminishes with maturity. A stronger preference for output stability results in less volatile bond rates for

bond as above. The return on the bond is then $\ln(B_t^m/B_{t-1}^m) \simeq -m(i_t^m - i_{t-1}^m)$.

similar reasons that it lowers the forecast error. Higher α is associated with less pronounced movements of the nominal short rate to counteract inflationary shocks, with the combined coefficient $\left(\frac{\lambda}{\alpha\gamma}\right)^2 \delta^2$ declining in α. While not shown here, more persistent shocks raise bond volatility. A positive value of ρ raises the tolerated inflation gap for a given aggregate supply shock with the partially offsetting effect of reducing the unconditional variance of $(u_t - u_{t-1})$.

Limited Information: The expression for the change in an m-period bond in the limited information case, $i_t^{m^{LI}}$, can be derived in a similar manner. Combining the policy rate at t dictated by the central bank's reaction function with predicted values of the short rate from (22) yields

$$\begin{aligned} i_t^{m^{LI}} &= \frac{1}{m}\left(i_t + i_{t+1/t}^{LI} + \ldots + i_{t+m-1/t}^{LI}\right) + \zeta_t^m \\ &= \frac{1}{m}\left(\pi_t^* + \frac{\lambda}{\alpha\gamma}\delta u_t + \frac{1}{\gamma} g_t + (m-1)\pi_{t/t}^*\right) + \zeta_t^m. \end{aligned} \tag{30}$$

The nominal component is pegged to a combination of the true inflation target, π_t^*, and the inferred value, $\hat{\pi}_{t/t}^*$, which is projected for $(m-1)$ periods of the bond. Taking first differences, the change in a bond's yield reflects true changes to the target as well as revisions to the perceived target,

$$i_t^{m^{LI}} - i_{t-1}^{m^{LI}} = \frac{1}{m}\left(\varepsilon_t + (m-1)(\pi_{t/t}^* - \pi_{t-1/t-1}^*) + \frac{\lambda}{\alpha\gamma}\delta(\hat{u}_t - \hat{u}_{t-1}) + \frac{1}{\gamma}(\hat{g}_t - \hat{g}_{t-1})\right). \tag{31}$$

The Kalman updating algorithm in (23) expresses $(\pi_{t/t}^* - \pi_{t-1/t-1}^*)$ in terms of current shocks, ε_t and $\hat{u}_t$, from which follows

$$\begin{aligned} \frac{\partial\left(i_t^{m^{LI}} - i_{t-1}^{m^{LI}}\right)}{\partial \varepsilon_t} &= 1 - \frac{m-1}{m}\phi \\ \frac{\partial\left(i_t^{m^{LI}} - i_{t-1}^{m^{LI}}\right)}{\partial \hat{u}_t} &= \frac{1}{m}\frac{\lambda}{\alpha\gamma}\delta + \frac{m-1}{m}(1-\phi). \end{aligned}$$

Comparing these to the equivalent partial derivatives for full information encapsulates the story of under- and over-reaction of bond yields to inflation news. The term structure under-reacts to permanent changes in the target (ε_t) relative to the full information scenario. As m grows, the reaction approaches the optimal gain $(1-\phi)$. In contrast, all maturities overreact to transitory disturbances. The bond yield moves with the transitory shock that entered the policy reaction function but also attributes $(1-\phi)$ of the shock to a revised estimate of the inflation target. The presence of the maturity reflects the number of periods for which the mistake is projected $(m-1)$ and the gain indicates the severity of the learning problem. The recursive nature of the problem complicates derivation of bond volatility so the reader is referred to Appendix E and the result presented here.

Proposition 3 *The volatility of returns to an m-period bond with limited information and serially uncorrelated errors is*

$$var(i_t^{m^{LI}} - i_{t-1}^{m^{LI}}) = A\sigma_\varepsilon^2 + \frac{1}{m^2}\left(B\delta^2\sigma_{\hat{u}}^2 + \frac{1}{\gamma^2}2\sigma_{\hat{g}}^2\right) + \sigma_\varsigma^2$$

where

$$A = \frac{1}{m^2}\left[[1+(m-1)(1-\phi)]^2 + \frac{((m-1)(1-\phi)\phi)^2}{1-\phi^2}\right] < 1$$

$$B = \left[\frac{\lambda}{\alpha\gamma} + (m-1)(1-\phi)\right]^2 + \left[\frac{\lambda}{\alpha\gamma} + (m-1)(1-\phi)^2\right]^2 + \left[\frac{((m-1)(1-\phi)^2\phi)^2}{1-\phi^2}\right] > 2\left(\frac{\lambda}{\alpha\gamma}\right)^2$$

For given σ_ε^2, $\sigma_{\hat{u}}^2$ *and* $\sigma_{\hat{g}}^2$ *and* $\phi\left(\frac{\sigma_\varepsilon^2}{\delta^2\sigma_{\hat{u}}^2}\right) \in (0,1)$

$$var(i_t^{m^{LI}} - i_{t-1}^{m^{LI}}) > var(i_t^{m^{FI}} - i_{t-1}^{m^{FI}}) \quad for \;\; m > 1.$$

Proof in Appendix E.

A and B are functions of maturity, the gain and the economy's parameters. A is less than one because bond markets systematically underreact to true changes in the target. Likewise, systematic overreaction to transitory disturbances results in B being greater than the corresponding coefficient in Proposition 2 ($\frac{1}{m^2}2\left(\frac{\lambda}{\alpha\gamma}\right)^2$). B also incorporates volatility in the bond return due to transitory shocks and is the reason that under- and over-reaction of the perceived target do not offset in the calculation.

As m rises, the effect of transitory shocks dies out and the movement in longer maturity bonds is dominated by revision in inflation expectations. Because the Kalman filter optimises to match this variance to the true variance of the target, the volatility of bonds under the two scenarios converge for large enough m.

As the signal-to-noise ratio increases (i.e., $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2} \to \infty$), volatility also converges whilst becoming absolutely larger. A approaches 1 and B is rising in $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}$ but less rapidly than $A\sigma_\varepsilon^2$.[14] Moving in the other direction, this implies that the ratio of bond volatility in the limited and full information scenario is greatest when the signal-to-noise ratio is weak (see Figure 5 below). The relative reduction in bond volatility achieved by communicating the target is greatest when that target is already close to being stable, an important message for central banks operating with an implicit inflation target. Finally, when $\sigma_\varepsilon^2 = 0$, there is no learning behaviour and the full and limited information variances coincide. In this situation, bond volatility is powerfully declining in maturity.

3.3.3 Two Sensitivity Puzzles

1. **Sensitivity of bond yields to monetary policy**

The model can shed light on a question that has been addressed empirically by previous authors. By how much should an m-period bond respond to movement in the policy controlled short rate? Cook and Hahn (1989) answer this question by regressing changes in the policy instrument on bond yield changes, which yields sizeable coefficients for the 1970s but not in later samples. Arguing that this reflected a larger anticipated element of target rate movements in recent years, Kuttner (2001) estimates the response of interest rates to *surprise* monetary policy actions

$$\Delta i_t^m = a + b_{1,m}(i_t - i_{t/t-1}) + e_t \tag{32}$$

where market expectations are derived using Fed futures contracts. This yields significant and surprisingly large estimates of b_1 (reproduced in Table 5 below). Even at a horizon as long as 30 years, a one percent rise in the Federal Funds rate is associated with a 17 basis point rise in bond yields. Ellingsen and Söderström (2004) estimate similar coefficients when measuring the unanticipated component of monetary policy as the change in the 3 month rate on days when the Fed funds rate was moved.

[14] As $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2} \to \infty$, $A \to 1$ but $B \to 2(\frac{\lambda}{\alpha\gamma})^2 + 4\frac{\lambda}{\alpha\gamma}(m-1) + 2(m-1)^2 > 2(\frac{\lambda}{\alpha\gamma})^2$. The two coefficients approach their limits at different speeds and the variances appropriately scale the coefficients such that bond volatility converges.

The coefficient b_1 can be given analytical form in the model, leading to the following proposition.

Proposition 4 *The coefficient in a regression of the change in an m-period bond on monetary policy surprises is larger when the inflation target is unobserved than when it is communicated. That is,*

$$b_{1,m}^{LI} = \frac{cov(\Delta i_t^m, i_t - i_{t/t-1})^{LI}}{var(i_t - i_{t/t-1})^{LI}} > \frac{cov(\Delta i_t^m, i_t - i_{t/t-1})^{FI}}{var(i_t - i_{t/t-1})^{FI}} = b_{1,m}^{FI}$$

for all $m > 1$ and $\frac{\sigma_\varepsilon^2}{\sigma_u^2} \in (0, \infty)$. Proof in Appendix F.

The intuition for the full information scenario is straightforward. Current shocks ε_t, u_t and g_t appear in both the nominal short rate forecast error and the change in bond yields and generate a covariance between the two. This covariance dies out over time as the effect of transitory shocks are scaled down in the bond calculation. Note, however, that because the covariance captures true policy innovations as well as transitory shocks, the coefficient can not be interpreted as the effect of pure monetary policy shocks.

With limited information and learning, we have seen that Δi_t^m and $i_t - i_{t/t-1}$ react not only to current shocks but incorporate a dependence on the historical realisation of shocks through $\pi_{t/t}^*$ (recall the polynomial lagged function in equation 24). This adds covariance terms. In addition, overreaction of long-run inflation expectations to transitory shocks ensures that the covariance between long interest rates and shocks to the policy reaction function does not die out as quickly.

2. **Volatility and Sensitivity of Forward Rates**

Forward rates also appear to be highly sensitive to inflation news. Gürkaynak et al. (2003) document two empirical facts about the behaviour of forward rates. First, the volatility of forward rates is not downward sloping with horizon; 10 and 15 year forward rates are as volatile as 2 year forward rates. Second, forward rates respond at very long horizons to current news – following inflation news surprises, the magnitude of the long response is often similar to that of one year rate.

Both types of behaviour are predicted by the model here and qualitatively this is due to time-variation in the inflation target. With unpredictable movements in the target, long-run inflation expectations are not anchored to a fixed point. Volatility of forward rates can be defined as the change in the forward rate at a fixed horizon from one quarter to the next in the same manner as Gürkaynak et al. (2003). The following expressions can be derived from the model:

$$\begin{aligned} i^{FI}_{t+j/t} - i^{FI}_{t+j/t-1} &= \pi^*_t - \pi^*_{t-1} = \varepsilon_t \\ var(i^{FI}_{t+j/t} - i^{FI}_{t+j/t-1}) &= \sigma^2_\varepsilon \quad \forall \ j > 1 \end{aligned}$$

$$\begin{aligned} i^{LI}_{t+j/t} - i^{LI}_{t+j/t-1} &= (1-\phi)(\pi_t - \pi^{LI}_{t/t-1}) = (1-\phi)k_t \\ var(i^{LI}_{t+j/t} - i^{LI}_{t+j/t-1}) &= (1-\phi)^2\sigma^2_k = \sigma^2_\varepsilon \quad \forall \ j > 1 \end{aligned}$$

These variances are not only constant over horizon j but share the same value when the gain is calibrated optimally to the signal-to-noise ratio. This occurs because the Kalman filter (the minimum variance estimator of the unobserved state variable) matches the second moment of the true and perceived targets. That is, in the univariate case $(1-\phi)^2\sigma^2_k = \sigma^2_\varepsilon$.

The response of forward rates to a one-standard-deviation inflation surprise is estimated by:

$$i_{t+j/t} - i_{t+j/t-1} = \alpha + b_{2,j}\frac{\pi_t - \pi_{t/t-1}}{stdev(\pi_t - \pi_{t/t-1})} + \epsilon_{t,j}$$

and yields the following coefficients:

$$\begin{aligned} b^{FI}_{2,j} &= \frac{\sigma^2_\varepsilon}{\sqrt{\sigma^2_\varepsilon + \delta^2\sigma^2_{\hat{u}}}} \quad \forall \ j > 1 \\ b^{LI}_{2,j} &= \sigma_\varepsilon \qquad \forall \ j > 1 \\ where \ \sigma_\varepsilon &> \frac{\sigma^2_\varepsilon}{\sqrt{\sigma^2_\varepsilon + \delta^2\sigma^2_{\hat{u}}}} \quad given \ \ \sigma^2_{\hat{u}} > 0 \end{aligned}$$

(see Appendix G). Both information scenarios deliver the qualitative property that the response of forward rates to macroeconomic news is constant over all horizons. However, the signal-extraction problem raises $b^{LI}_{2,j}$ to exceed $b^{FI}_{2,j}$ for all j for the same reasons that the coefficient on monetary policy innovations was higher. This is closely related to the empirical finding of Diebold, Rudebusch, and Aruoba (2003) that inflation surprises boost the level factor in a latent factor model of the yield curve. This is consistent with the mechanism in the model here in which inflation surprises contain information which prompt revision of the perceived target and long-run inflation expectations.

4 Quantitative Implications

4.1 Calibration

Having shown for a simple case that time-variation in the inflation target and learning contribute to the volatility and sensitivity of bond yields, what are the relative magnitudes involved? To answer this I present calibrations from the full version of the model in which bond markets use both observation equations to infer the values of unobserved state variables and transitory shocks are serially correlated. Plausible parameter choices yield predictions from the model of approximately the right magnitude. Such a calibration exercise is useful to assess the relative contribution of the mechanisms in the model, particularly for the long maturity interest rates to which the model is better suited. Variance of the inflation target is used to match the observed volatility in bond returns at the long end of the yield curve. The factors captured by the model are obviously not the only sources of bond volatility but Ang and Bekaert (2004) find that those that are excluded here, namely time variation in term premia and the real interest rate, account for very little in empirical variance decompositions.

Structural parameters for a simple DSGE model are suggested by Clarida, Gali, and Gertler (2000): an elasticity of inflation with respect to the output gap of 0.3 in the Phillips curve and a one-to-one relationship between the output gap and the real interest rate in the aggregate demand equation. To match the persistence in inflation and output they assume highly serially correlated disturbances in their simulations ($\rho = \mu = 0.9$) which imply that a shock has a half-life of over six quarters. This is unlikely to be appropriate here as the non-stationary inflation target accounts for much of the persistence in inflation. I instead assume a common persistence parameter of 0.5 implying a half-life of just over one quarter.[15]

[15] Beechey, Carlsson and Österholm (2004) use the decomposition suggested by this model to re-examine the time series properties of transitory shocks to the economy once a random walk in the inflation target has been filtered out. The serial correlation in the residuals is substantially lower and between 0.4 to 0.7.

Table 1: Parameter Calibrations

Structural Parameters		Variances	
β (Consumption discount rate)	0.99	$\sigma^2_{\hat{u}}$	1.0
λ (Elasticity of $\pi_t - \pi^*_t$ wrt y_t)	0.3	$\sigma^2_{\hat{g}}$	0.7
γ (Elasticity of output wrt real interest rate)	1	σ^2_{ε}	$0, 0.2, 0.35, 0.7$
α (Preference in central bank's loss function)	0.5		
ρ, μ (Persistence of transitory shocks)	0.5		

Notes: Variances pertain to annualised quarterly observations. That is, $z_t = 400 * (\ln Z_t - \ln Z_{t-1})$.

Clarida et al. (2000) do not suggest variances for the transitory shocks in their model. Rudebusch (2002) estimates a partially backward-looking variant of a new Keynesian model, also assuming a constant steady state, and reports estimated variances of serially uncorrelated aggregate supply and demand shocks. Expressed as shocks to annualised quarterly inflation and the level of the output gap, these variances are approximately 1 and 0.7 respectively. Whilst it is not clear that these are the correct variances for the model at hand, varying them within plausible ranges does not materially affect the implications of the calibration exercise. Note that by assuming $\rho = \mu = 0.5$, the unconditional variances σ^2_u and σ^2_g are one third larger than $\sigma^2_{\hat{u}}$ and $\sigma^2_{\hat{g}}$ respectively.

There are few estimates of the variance of innovations to the inflation target. Smets and Wouters (2003) estimate the quarterly innovation variance for a random walk inflation target to be 0.055 (median estimate) for the U.S. between 1973 and 2003 and 0.099 for the Euro-area. Kozicki and Tinsley (2003) find a similar estimate of 0.044 using U.S. data from 1960, although they use a dummy variable to account for the changes in the early Volcker years.

To put these estimates in perspective, a quarterly innovation variance of 0.05 (annualised variance of 0.8) implies a standard deviation of 1.4 percentage points in the inflation target over one decade. Likewise, an innovation variance of 0.01 (annualised variance approximately 0.2) implies a standard deviation of 0.7 percentage points over a decade. The basic parameter choices are summarised in Table 1.

4.2 Volatility of Bond Returns

Table 2 shows the variance of quarterly changes in constant maturity bonds in the United States from 1981 to 2004. The volatility of bond returns declined substantially in the 1990s

Table 2: Volatility of returns on U.S. Treasuries, 1981Q1 to 2004Q3

Maturity (*years*)	1981Q1 - 2004Q3	1981Q1 - 1989Q4	1990Q1 - 2004Q3
1	0.74	1.51	0.27
2	0.72	1.35	0.34
5	0.61	1.06	0.33
10	0.46	0.82	0.24
20*	–	–	–
30	0.37	0.67	0.15**

Notes: Volatility is calculated as the variance of the quarter-to-quarter change in the reported bond yield. Data are end quarter observations March, June, September, December. *Missing data 1987 to 1994. ** Missing data March 2002 to end of sample. Source: Board of Governors H.15 Database, selected constant maturity treasury bonds.

Table 3: Calibrated volatility of bond returns, Full Model

Maturity	Constant $\sigma^2_\epsilon = 0$	Moderate $\sigma^2_\varepsilon = 0.35$		High $\sigma^2_\varepsilon = 0.7$		Low $\sigma^2_\varepsilon = 0.2$	
(*years*)		*FI*	*LI*	*FI*	*LI*	*FI*	*LI*
1	0.61	0.95	1.25	1.31	1.68	0.81	1.05
2	0.17	0.52	0.74	0.87	1.15	0.37	0.55
5	0.03	0.38	0.48	0.73	0.86	0.23	0.31
10	0.01	0.36	0.41	0.71	0.78	0.21	0.25
20	0.00	0.35	0.38	0.71	0.74	0.20	0.22
30	0.00	0.35	0.37	0.70	0.72	0.20	0.22

Notes: The data in the table are generated with Monte Carlo simulations using 1000 draws of the full model economy observed for 75 years. Reported numbers are the mean of the variances over all simulations.

relative to the preceding decade across all maturities but behaves broadly as expected in both periods with volatility declining as term increases.

Simulated volatility for the full model (bond markets learn through both short rate and inflation forecast errors) is shown in Table 3. Four calibrations of the variance of the inflation target (σ^2_ε) are shown, labeled *constant*, *moderate*, *high* and *low*. The latter three are chosen to roughly correspond to the volatility at the long end of the curve for the samples in Table 2.

Results for a constant inflation target are shown in column 2. Volatility declines rapidly with maturity, leaving very little variance in even a 5 year bond. Observed bond yields exhibit substantially more volatility at long maturities and the benchmark of a constant inflation target appears a poor approximation. Figure 2 plots this, alongside predicted values from the model with a time-varying target.

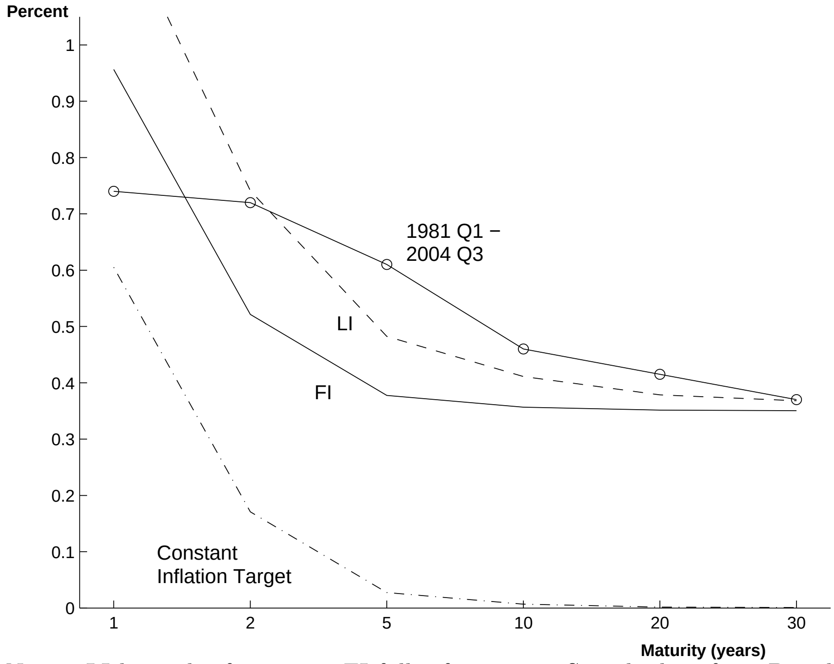
In the subsequent columns of Table 2, volatility in the full information (FI) and limited information (LI) scenarios are reported for the three calibrations of σ_ϵ^2 (*moderate*, *high* and *low*). There are several points worth making here. First, the contribution of a time-varying inflation target to bond volatility is substantial. For all calibrations, it is movement in the target that is responsible for bond volatility once transitory shocks have died out. Second, learning contributes additional volatility compared to the full information counterfactual, between 8 and 13 basis points for a 5-year bond and 4 and 7 basis points for a 10-year bond. For the high scenario, this represents about one tenth of total volatility in the ten year bond; for the low scenario one fifth. Third, the wedge created by learning diminishes with maturity. This occurs because as maturity grows, revision of long-run inflation expectations comes to dominate movement in the bond and the optimal learning strategy calibrates the variance of revisions to the variance of changes in the true target.

Figures 2, 3 and 4 plot the values in Table 3 against actual volatility for the three sample periods. In Figure 3 it can be seen that the expectations hypothesis paired with a time-varying inflation target does a good job of matching the volatility of the term structure in the 1980s. For the latter half of the sample the model does a relatively poor job of mimicking the short end of the yield curve, as short term interest rates predicted by the model are more variable than in the data. In part this reflects that the optimal monetary policy reaction function in the model does not incorporate an interest rate smoothing term at a time when policy movements have become smoother.[16] The model does a better job for longer maturities and it is clear that some degree of time variation is needed to match the volatility in long bond returns.

These calibrations have assumed that variation in the inflation target accounts for most of the movement in bond yields over time. If variation in the term premium does account for some of the movement, this does not lower the estimated contribution made by learning. For lower values of σ_ε^2 and a weaker signal-to-noise ratio, the *relative* contribution of learning to volatility rises. This can be seen more clearly in Figures 5 which plots the ratio of the variance of a 10-year bond between the limited and full information scenarios for a richer range of σ_ε^2

[16]Introducing an ad hoc smoothing term in the monetary policy reaction function lowers the immediate response of the policy-controlled short rate to current shocks and thus lowers the variance in 1 and 2 year bond changes with little effect on longer maturities.

Figure 2: Volatility of returns, 1981Q1 to 2004Q3

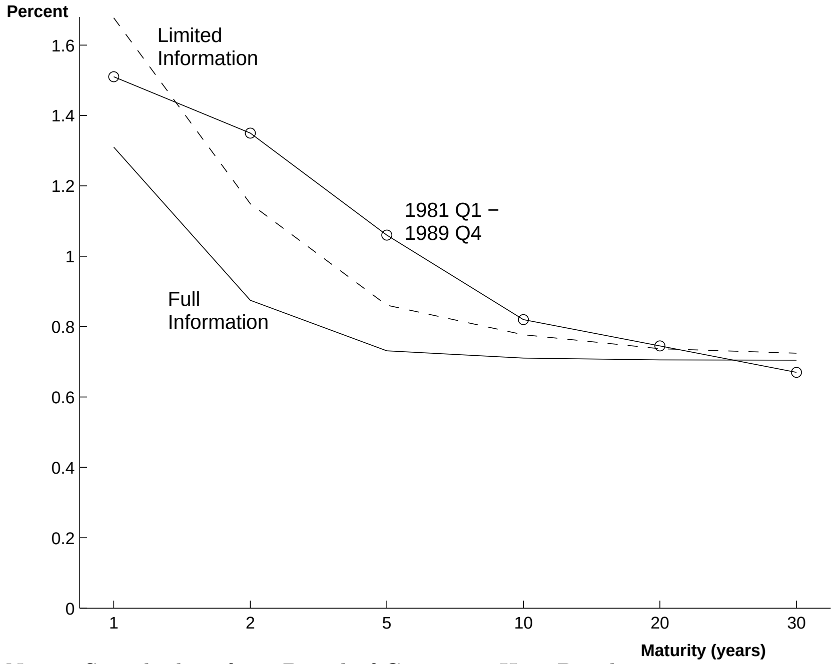


Notes: LI limited information, FI full information. Sample data from Board of Governors H.15 Database, constant maturity treasuries (missing data for 20 year bond). Calibrated values from Table 3, columns 2, 3 and 4.

calibrations. The ratio of variances is greatest when the signal-to-noise ratio is weakest.

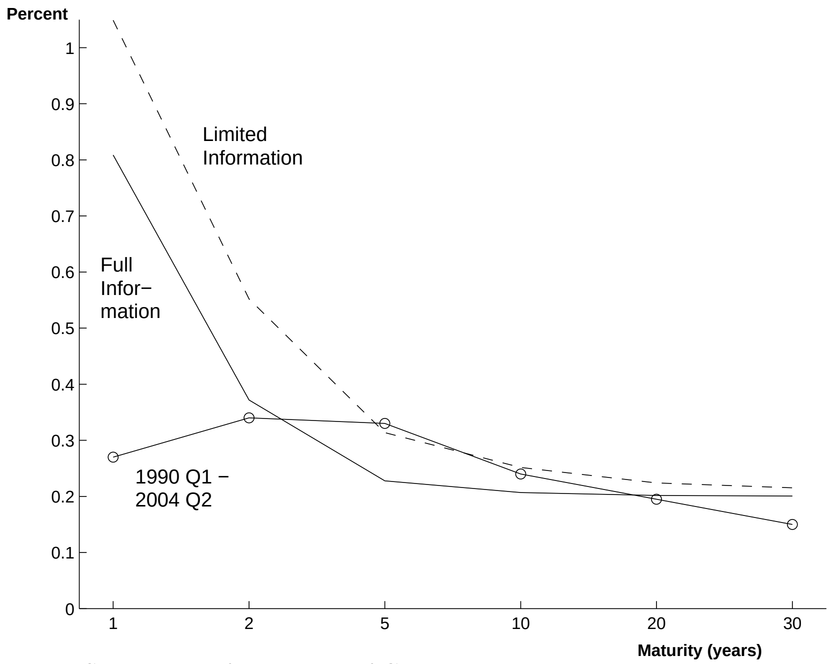
The results at the long end of the term structure are not particularly sensitive to alternative parameter calibrations. Lowering the central bank's preference for output stability (α) to 0.2 raises the volatility of short interest rates (1 and 2 years) because of the central bank's greater willingness to create output deviations to restore inflation to the target but not long rates. Lowering the persistence of the transitory shocks to $\rho = \mu = 0.3$ lowers volatility at all maturities, although again, the effect is most pronounced at the short end when transitory shocks feature more heavily in interest rate forecasts. For the 10-year bond, the difference is only a matter of 2 basis points. When the common serial correlation parameter is raised to 0.8, persistence raises the variance of a 10-year bond to 0.11 even with a constant inflation target. However, this degree of autocorrelation seems implausibly high, causing the variance of 1 and 2 year bonds to reach 3.8 and 1.9 respectively, grossly inconsistent with the data in Table 2. Lastly, lowering the elasticity of output with respect to the real interest rate (γ) from 1 to 0.5, so that the central bank needs to move interest rates by more to achieve the same effect on inflation, has the effect of significantly raising volatility at the short end (1 and

Figure 3: Volatility of returns, 1981Q1 to 1989Q4



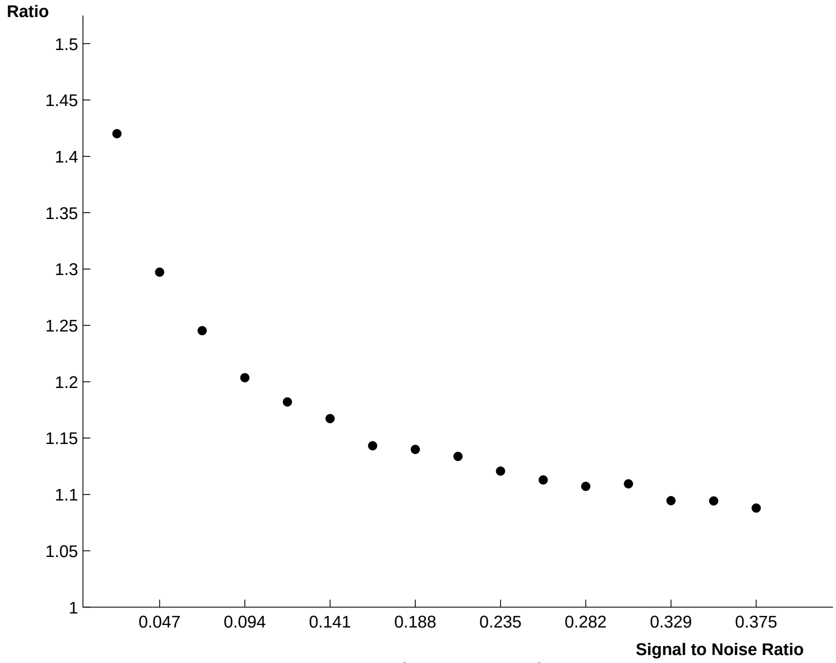
Notes: Sample data from Board of Governors H.15 Database, constant maturity treasuries (missing data for 20 year bond). Calibrated values from Table 3, columns 5 and 6 for $\sigma_\varepsilon^2 = 0.7$.

Figure 4: Volatility of returns, 1990Q1 to 2004Q3



Notes: Sample data from Board of Governors H.15 Database, constant maturity treasuries (missing data for 20 year bond). Calibrated values from Table 3, columns 7 and 8 for $\sigma_\varepsilon^2 = 0.2$.

Figure 5: Ratio of volatility in 10-year bond returns



Notes: The graph shows the ratio of volatility of quarter-to-quarter changes in a 10-year bond between the limited and full information cases for the range $\sigma_\varepsilon^2 \in [0.05, 0.80]$. Generated using Monte Carlo simulations as in Table 3.

2 year bonds) without imparting much additional variance at or beyond the 10-year bond.

In Section 3 it was noted that the policy short rate is a noisier signal of the inflation target in this model. Calibrating the model for the case in which bond markets purely learn through inflation surprises lowers volatility slightly for the limited information scenario (see Table 4). The differences, however, are minor and only a mater of 1 to 2 basis points at the long end.

4.3 Two Sensitivity puzzles

We now return to the model's quantitative predictions about the sensitivity puzzles discussed in Section 3.

1. **Sensitivity of bond yields to monetary policy innovations**

Table 5 reproduces the response of long interest rates to monetary policy surprises estimated by other authors. Ellingsen and Söderström (2004) and Craine and Martin (2004) note that the size of the coefficients reflects a combination of exogenous and endogenous monetary policy shocks, that is, target changes and transitory shocks that enter the monetary policy reaction function respectively. For this reason they should not be viewed as measuring the

Table 4: Calibrated volatility of bond returns (π observation equation only)

Maturity	$\sigma_\epsilon^2 = 0$	$\sigma_\epsilon^2 = 0.35$		$\sigma_\epsilon^2 = 0.7$		$\sigma_\epsilon^2 = 0.2$	
(*years*)		*FI*	*LI*	*FI*	*LI*	*FI*	*LI*
1	0.61	0.95	1.19	1.31	1.58	0.81	1.00
2	0.17	0.52	0.70	0.87	1.08	0.37	0.51
5	0.03	0.38	0.46	0.73	0.83	0.23	0.30
10	0.01	0.36	0.40	0.71	0.76	0.21	0.24
20	0.00	0.35	0.37	0.71	0.73	0.20	0.22
30	0.00	0.35	0.37	0.70	0.72	0.20	0.21

Notes: The data in the table are generated with Monte Carlo simulations using 1000 draws of the full model economy observed for 75 years. Reported numbers are the mean of the variances over all simulations.

Table 5: Response of interest rates to Fed Funds rate surprises

Maturity	Kuttner		Ellingsen & Söderström	
(*years*)	Coefficient	standard error	Coefficient	standard error
1	0.72	(0.08)	0.83	(0.09)
2	0.61	(0.06)	0.68	(0.10)
5	0.48	(0.04)	0.50	(0.11)
10	0.32	(0.03)	0.29	(0.11)
20	–	–	–	–
30	0.19	(0.02)	0.17	(0.09)

Notes: Coefficient estimates with standard errors in parentheses reproduced from Kuttner (2001) and Ellingsen and Söderström (2004).

pure effect of monetary policy. The endogenous component can reflect shocks that are commonly observed by the central bank and bond markets or asymmetrically-held information that is revealed through inflation or monetary policy.

Calibrated values of the coefficient $b_{1,n}$ are shown in Table 6 for a constant inflation target and the moderate and low values of target innovation variance likely to describe the period after 1989 (all other parameters are the same as those outlined in Table 1). The simulated coefficients are of a similar magnitude to those estimated by Kuttner (2001) and Ellingsen and Söderström (2004), especially for longer maturities, although at times both $b_{1,m}^{LI}$ and $b_{1,m}^{FI}$ fall within one standard deviation of the estimates.

The coefficients in Table 6 decline with maturity as the averaging inherent in the expectations hypothesis reduces the covariance between long and short rates. For a constant inflation target (Table 6 column 2), long maturity bonds are anchored to a fixed point and shocks to the monetary policy reaction function have little effect at long horizons. Thus the

Table 6: Calibrated response of interest rates to Fed Funds rate surprises

Maturity	$\sigma_{\varepsilon}^2 = 0$	$\sigma_{\varepsilon}^2 = 0.35$		$\sigma_{\varepsilon}^2 = 0.2$	
(years)	$\hat{b}_{2,m}^{FI} = \hat{b}_{2,m}^{LI}$	$\hat{b}_{2,m}^{FI}$	$\hat{b}_{2,m}^{LI}$	$\hat{b}_{2,m}^{FI}$	$\hat{b}_{2,m}^{LI}$
1	0.47	0.55	0.65	0.52	0.62
2	0.25	0.36	0.51	0.32	0.45
5	0.10	0.23	0.41	0.18	0.34
10	0.05	0.19	0.38	0.13	0.31
20	0.03	0.17	0.36	0.11	0.29
30	0.02	0.16	0.36	0.10	0.28

Notes: Calibrated values generated with Monte Carlo simulations as in Tables 3 and 4.

coefficients quickly decline to zero. Comparing to columns 3 and 5, the counterfactual case of a communicated, time-varying inflation target raises the estimates substantially. This occurs because long-run inflation expectations are revised in accordance with observed changes in the inflation target.

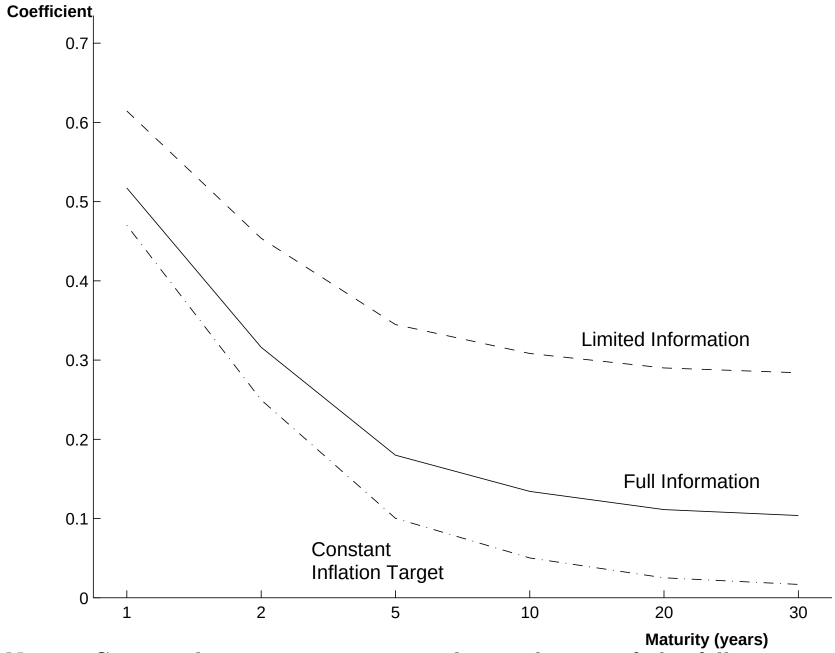
The effect of limited information can be seen by comparing to columns 4 and 6. Figure 6 also plots the data in the table for for $\sigma_{\varepsilon}^2 = 0, 0.2$. Constant-gain learning substantially increases the coefficients that should be expected from a regression of long interest rates on monetary policy surprises. This is especially so for longer interest rates whose movements are dominated by revisions in inflation expectations. Because such revisions respond to current transitory shocks that also enter the policy reaction function, the covariance between surprise short rate movements and long interest rates remains large with maturity.

When bond markets are confined to learning purely through the inflation observation equation, the predicted regression coefficients are 3 to 5 basis points lower depending on maturity but still double those of the full information counterfactual.

2. **The Volatility and Sensitivity of Forward Rates**

Gürkaynak et al. (2003) report that the volatility of forward rates (standard deviation of quarterly changes) between 1990 to 2002 is 1.3 percent at the 1 year to horizon and declines smoothly to 1 percent point at 15 years. For the calibrations considered in this paper, the moderate innovation variance corresponds to a standard deviation, σ_{ε}, of 0.59 and the low to 0.44 which is around half of the reported volatility. Introducing serially correlated errors imparts slightly more volatility to near-horizon forward rates, consistent with their empirical

Figure 6: Response of interest rates to monetary policy surprises



Notes: Generated using 1000 monte carlo simulations of the full economy observed for 75 years for $\sigma_\varepsilon^2 = 0.2$. Values plotted are the mean of coefficients over all simulations from Table 6, columns 2, 5 and 6.

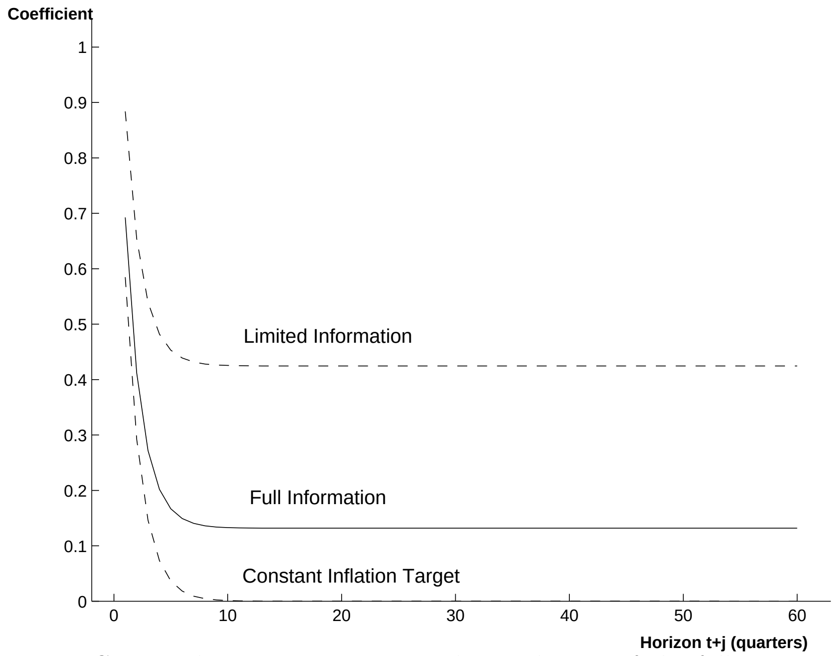
findings.[17] The predictions of the model contrast strongly with those of a partly backward-looking model with constant steady state such as Rudebusch (2002). While such models generate substantial persistence in inflation, the purely transitory nature of the shocks results in a strongly downward sloping volatility profile.

Turning to the calibrated regression coefficients, Figure 7 plots the coefficients $b_{2,j}^{FI}$ and $b_{2,j}^{LI}$ for the low calibration $\sigma_\varepsilon^2 = 0.2$ and for a constant inflation target. The coefficients represent the average movement of forward rates associated with a one standard deviation inflation surprise. With a constant target, the inflation surprise consists only of transitory shocks which are not factored into long-run inflation expectations. Thus the coefficient quickly reaches zero. With a random-walk inflation target and full information, the coefficient does not decay to zero because of the covariance between changes in the target and revision of inflation expectations.

[17] This can be seen most easily for the full information case:

$$var(i_{t+j/t}^{FI} - i_{t+j/t-1}^{FI}) = \sigma_\varepsilon^2 + \left(\rho + \frac{\lambda}{\alpha\varphi}(1-\rho)\right)^2 \left(\delta^2\rho^{2j}\sigma_{\hat{u}}^2 + \mu^{2j}\frac{\sigma_{\hat{g}}^2}{\varphi^2}\right) \quad \text{for all } j \geqslant 1 \tag{33}$$

Figure 7: Response of forward rates to inflation surprises



Notes: Generated using 1000 monte carlo simulations of the full economy observed for 75 years for $\sigma_\varepsilon^2 = 0.2$. Values plotted are the mean of coefficients over all simulations.

With constant-gain learning and a weak signal-to-noise ratio (low σ_ε^2), the reaction 20 quarters ahead (0.43) is three times as great as the reaction with full information (0.13). This occurs because long-run inflation expectations co-vary with permanent as well as transitory shocks, substantially increasing the predicted coefficient from such a regression. As σ_ε^2 rises, the ratio of $b_{2,j}^{LI}$ to $b_{2,j}^{FI}$ declines but does not fall below two for the calibrations considered here. What this analysis makes clear is that it is time-variation and persistence in the inflation target that replicates the qualitative finding that forward rates respond to inflation surprises at long horizons. Asymmetric information and inference substantially increase the magnitude.

4.4 Extensions

1. **Learning with the Wrong Gain**

In the analysis above, the rate at which bond market participants learn about the inflation target is assumed to be calibrated to the true signal-to-noise ratio in the economy. However, agents may be learning at the wrong rate for a number of reasons including insufficient information about the true gain, an attempt to learn more quickly the character of a new policy regime following a change, or a lack of credibility in an announced inflation target. Small deviations from the optimal gain can have sizeable implications for volatility. With faster learning, agents adjust to permanent structural shocks more rapidly but also incorporate more transitory shocks into their estimate of the state variables. For example, building upon the structurally stable model posited by Orphanides and Williams (2003), Beechey (2004) shows that a small and positive gain imparts sufficient volatility to long interest rates to reject the null hypothesis in a Shiller style test of excess volatility.

2. **Inflation Targeting**

Proponents of inflation targeting sometimes claim greater financial market stability as one of the potential benefits of such a policy. The mechanism described in this paper suggests that this benefit should arise through two channels - by stabilising the nominal target and by communicating its value. One implication is that conditional on the mean and variance of macroeconomic shocks, long term interest rates should exhibit less volatility under inflation targeting regimes.

Bond volatility should offer a test of the success of such targets in anchoring long-run expectations. This is easier said than done, since it is import to control for differences in the magnitude of macroeconomic shocks and idiosyncratic differences in term premium variation could confound the exercise. In addition, some inflation-targeting central banks operate with loosely defined targets that may not differ functionally from the implicit inflation targeting practised in the US – Australia, for example, has a target band with loosely defined medium-term horizon. Furthermore, some targeting regimes may not yet have earned sufficient credibility to affect bond market outcomes.

Another implication of the model is that when central banks successfully communicate inflation goals, forward rates should become less sensitive to inflation news. This is borne out in the empirical work of Gürkaynak, Sack, and Swanson (2003) for the United Kingdom where the Bank of England has had some success stabilising inflation expectations. Gauging how much of this is due to communication about the target versus how much is due to stabilising the target itself could be addressed with a framework like the one proposed in this paper.

5 Conclusions

The volatility and sensitivity of long interest rates are closely related. As previous authors have noted, non-stationary nominal short rates are more likely to reconcile observed bond volatility with the expectations hypothesis of the term structure than mean-reverting short rates. This paper has built non-stationarity into a standard forward-looking model via the inflation target then introduced an asymmetric information problem in which bond markets learn adaptively about time-varying policy preferences.

Asymmetric information and learning are key to understanding the sensitivity of long interest rates. Revelation of asymmetric information via noisy signals results in long-run inflation expectations being adjusted in response to both permanent and transitory shocks. This generates substantial co-movement between long interest rates and current transitory shocks and offers an explanation for the sensitivity of bonds as long as 30 years to current monetary policy and the reaction of forward rates at very long horizons to inflation news.

The asymmetric information problem also imparts additional volatility to bond returns. The source of this is the tendency of the perceived inflation target to overreact to transitory shocks in the economy and confound them with permanent shocks to the inflation target. Time variation in the inflation target is the main source of volatility in the model but learning adds to the ability of the model to match the observed volatility of returns. Calibration of the model suggests that for 5- and 10-year bonds respectively, a quarter to a fifth of volatility can be attributed to the inference problem. Notably, this channel is potent even when the degree of time variation in policy goals is small.

Worthwhile extensions of the analysis in this paper include enriching the model to include backward-looking elements and studying the implications of feedback of interest rate volatility into aggregate demand. The information assumptions have meant that strategic aspects of expectations formation have not been considered but these could be important in related frameworks.

The framework proposed in this paper can be used to address such questions as the source of the compression in bond volatility observed in much of the OECD during the mid-1990s, and the decline in the amplitude of forecast errors of the nominal short rate (Swanson, 2004). Whilst some have answered that the amplitude of shocks arriving in the economy has compressed, others have pointed to a shift toward explicit or implicit inflation targeting in certain countries and to improvements in central bank transparency and communication. All three channels are at work in the model presented here and the view that reducing the degree of time-variation in policy preferences is a factor finds support. The results also imply that a central bank operating a relatively stable yet uncommunicated target can ameliorate the inference problem and further reduce financial market volatility by regularly announcing its policy goals.

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2004): "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," Sveriges Riksbank.

Ang, A., and G. Bekaert (2004): "The Term Structure of Real Rates and Expected Inflation," Unpublished manuscript, Columbia Business School.

Beechey, M. (2004): "Limited Information, Learning and Bond Volatility," Unpublished manuscript, U.C. Berkeley.

Beechey, M., M. Carlsson, and P. Österholm (2004): "Decomposing Inflation: Does Persistence Really Stem From Private Agent's Behavior?," Unpublished manuscript, Uppsala University.

Calvo, G. (1983): "Staggered Prices in a Utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.

Clarida, R., J. Gali, and P. Gertler (1999): "The Science of Monetary Policy: A New Keynsian Perspective," *Journal of Economic Literature*, 37(4), 1661–1707.

——— (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115(1), 147–180.

Cogley, T., M. Morozov, and T. Sargent (2003): "Bayesian Fan Charts for U.K. Inflation: Forecasting and Sources of Uncertainty in an Evolving Monetary System," CFS Working Paper Series 2003/44.

Cook, T., and T. Hahn (1989): "The Effect of Changes in the Federal Funds Rate Target on Market Rates in the 1970s," *Journal of Monetary Economics*, 24(3), 331–351.

Craine, R., and V. Martin (2003): "Monetary Policy Shocks and Security Market Responses," mimeo U.C. Berkeley.

Cushing, M., and L. Ackert (1994): "Interest Rate Innovations and the Volatility of Long-Term Bond Yields," *Journal of Money, Credit and Banking*, 26(2), 203–217.

Diebold, F., G. Rudebusch, and S. B. Aruoba (2003): "The Macroeconomy and the Yield Curve: A Non-structural Analysis," Center for Financial Studies Working Paper No. 2003/31.

Ellingsen, T., and U. Söderström (2001): "Monetary Policy and Market Interest Rates," *American Economic Review*, 91(5), 1594–1607.

——— (2004): "Why are Long Rates Sensitive to Monetary Policy?," Sveriges Riksbank Working Paper Series No. 160.

Flavin, M. (1981): "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Literature," *Journal of Political Economy*, 91, 929–56.

Fuhrer, J. C. (1996): "Monetary Policy Shifts and Long-Term Interest Rates," *Quarterly Journal of Economics*, 111(4), 1183–1209.

Fuhrer, J. C., and G. Moore (1995): "Inflation Persistence," *Quarterly Journal of Economics*, 110(1), 127–159.

Gali, J. (2003): *Advances in Economic Theory*chap. New perspectives on monetary policy, inflation and the business cycle, pp. 151–197. Cambridge University Press.

Gürkaynak, R., B. Sack, and E. Swanson (2003): "The Excess Sensitivity of Long Term Interest Rates: Evidence and Implications for Macroeconomic Models," mimeo, Federal Reserve Board of Governors.

Hamilton, J. (1994): *Time Series Analysis*. Princeton University Press.

Honkapojha, S., and K. Mitra (2003): "Learning with bounded memory in stochastic models," *Journal of Economic Dynamics and Control*, 27, 1437–1457.

Kozicki, S., and P. Tinsley (2003): "Permanent and Transitory Policy Shocks in an Empirical Macro Model with Asymmetric Information," CFS Working Paper No. 2003/41.

Kuttner, K. (2001): "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market," *Journal of Monetary Economics*, 47(3), 523–544.

Levin, A., and J. Piger (2002): "Is Inflation Persistence Intrinsic in Industrial Countries?," Working Paper 2002-023E, Federal Reserve Bank of St Louis.

Marsh, T., and R. Merton (1986): "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," *American Economic Review*, 76(3), 483–503.

Mishkin, F. (1992): "Is the Fisher Effect for real?: A reexamination of the relationship between inflation and interest rates," *Journal of Monetary Econonmics*, 30(2), 195–215.

Orphanides, A., and J. C. Williams (2003): "Imperfect Knowledge, Inflation Expectations and Monetary Policy," NBER Working Paper 9884.

Preston, B. (2002): "Learning About Monetary Policy Rules when Long-Horizon Expectations Matter," Unpublished, Princeton University.

Romer, C., and D. Romer (2000): "Federal Reserve Information and the Behavior of Interest Rates," *American Economic Review*, 90(3), 429–457.

Rudebusch, G. (1995): "Federal Reserve Interest Rate Targeting, Rational Expectations and the Term Structure," *Journal of Monetary Economics*, 35, 245–274.

——— (2002): "Assessing nominal income rules for monetary policy with model and data uncertainty," *Economic Journal*, 112, 402–432.

Rudebusch, G., and T. Wu (2003): "A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy," FRBSF Working Paper 2003-17.

Sargent, T. (1979): *Macroeconomic Theory*. Academic Press, New York.

Shiller, R. (1979): "The Volatility of Long-term Interest Rates and Expectations Models of the Term Structure," *Journal of Political Economy*, 87(6), 1190–1219.

Singleton, K. (1980): "Expectations Models of the Term Structure and Implied Variance Bounds," *Journal of Political Economy*, 88(6), 1159–76.

Smets, F., and R. Wouters (2003): "Comparing Shocks and Frictions in US and Euro Area Business Cycles: A Bayesian DSGE Approach," *Journal of Applied Econometrics*, forthcoming.

Söderlind, P. (1999): "Solution and Estimation of RE Macromodels with Optimal Policy," *European Economic Review*, 43(4-6), 813–23.

Swanson, E. T. (2004): "Federal Reserve Transparency and Financial Market Forecasts of Short-term Interest Rates," Board of Governors, Federal Reserve.

Wallace, M., and J. Warner (1993): "The Fisher Effect and the Term Structure of Interest Rates: Tests of Cointegration," *Review of Economics and Statistics*, 75(2), 320–324.

Appendix A - Microeconomic foundations

Aggregate Demand: An infinitely-lived, representative household chooses C_t (consumption), N_t (household size) and B_t (assets) to maximise lifetime utility:

$$\max_{C_t,N_t,B_t} \mathsf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

s.t. $P_t C_t + (1+i_t)^{-1} B_t = B_{t-1} + W_t N_t$ for all t

where C_t and P_t are constant elasticity of substitution combinations of goods over measure 1. The first order conditions of the standard Lagrangian for this problem yield the consumption Euler equation:

$$1 = \mathsf{E}_t \left\{ \beta(1+i_t) \left[\frac{C_{t+1}}{C_t} \right]^{-\sigma} \frac{P_t}{P_{t+1}} \right\}.$$

Log-linearising the Euler equation around the steady state yields

$$c_t = \mathsf{E}_t c_{t+1} - \frac{1}{\sigma} \left[i_t - \mathsf{E}_t \pi_{t+1} \right]$$

where lower case letters denote log deviations. Paired with the market clearing condition $c_t = y_t$ and rewritten in terms of the output gap, $x_t = y_t - y_t^P$ (the deviation of output from the flexible price "potential" level y_t^P) this yields the aggregate demand equation in the text. (This can also encompass exogenously evolving government spending.) The steady state implies a constant real interest rate around which the Euler equation is linearised.

Aggregate Supply: Following Adolfson, Laséen, Lindé, and Villani (2004), the market for final goods is perfectly competitive and the production function of the final good firm transforms intermediate inputs into final output according to

$$Y_t = \left[\int_0^1 Y_{i.t}^{\frac{1}{\lambda_t}} \right]^{\lambda_t}, 1 \leq \lambda < \infty.$$

Profit maximisation leads to a relationship between the average price level of final goods, P_t, and the prices of intermediate goods, $P_{i,t}$:

$$P_t = \left[\int_0^1 P_{i.t}^{\frac{1}{1-\lambda_t}} \right]^{1-\lambda_t}. \tag{34}$$

A continuum of intermediate firms, each producing a differentiated good, faces monopolistic competition. As in Calvo (1983), the probability that a firm can re-optimise its price in a given period is constant and equal to $(1-\xi_p)$. For a given firm i, its re-optimised price is $P_{i,t}^{new}$. If a firm does not re-optimise then its price at $t+1$ is indexed according to $P_{i,t+1} = \pi_t^{\gamma_p} (\pi_t^*)^{1-\gamma_p} P_{i,t}^{new}$ where $\gamma_p \in [0,1]$. Thus the price the firm can charge if it has not re-optimised in j periods is $(\pi_t \pi_{t+1} \ldots \pi_{t+j-1})^{\gamma_p} \left(\pi_{t+1}^* \pi_{t+2}^* \ldots \pi_{t+j}^* \right)^{1-\gamma_p} P_{i,t}^{new}$.

The representative firm faces the following optimisation problem when setting its price,

$$\max_{P_{i,t}^{new}} \mathsf{E}_t \sum_{j=0}^{\infty} (\beta\xi)^j \, \upsilon_{t+j} \left[\begin{array}{c} \left((\pi_t \pi_{t+1} \ldots \pi_{t+j-1})^{\gamma_p} \left(\pi_{t+1}^* \pi_{t+2}^* \ldots \pi_{t+j}^* \right)^{1-\gamma_p} P_{i,t}^{new} \right) Y_{i,t+j} \\ -MC_{i,t+j} Y_{i,t+j} - MC_{i,t+j} z_{t+j} \phi \end{array} \right]$$

where $\beta \upsilon_{t+j}$ is the stochastic discount factor between periods t and $t+j$ used to discount profits, $Y_{i,t+j}$ is the output of the i^{th} intermediate firm, $MC_{i,t+j}$ its real marginal cost and z_{t+j} a permanent technology shock in the intermediate goods production function.

Smets and Wouters (2003) and Adolfson, Laséen, Lindé, and Villani (2004) show that the first order condition of the optimisation problem combined with the average price level in (34) yields a log-linearised Phillips curve showing the relationship between inflation, real marginal cost. A special case is when the inflation target follows a random walk:

$$\pi_t - \pi_t^* = \frac{\beta}{1+\gamma_p\beta} (\mathsf{E}_t \pi_{t+1} - \pi_t^*) + \frac{\gamma_p}{1+\gamma_p\beta} (\pi_{t-1} - \pi_t^*) + \frac{(1-\xi_p)}{\xi_p} \frac{(1-\beta\xi_p)}{1+\gamma_p\beta} (mc_t + \lambda_t)$$

where π_t and mc_t are log deviations from their respective steady state values. In this form, the deviation of inflation from the target depends on past and expected future inflation as well as marginal cost. Assuming that $mc_t = \beta_1 x_t + u_t$ where u_t can be interpreted as a cost-push shock or as a markup shock as in Gali (2003) and that $\gamma_p = 0$ (i.e., non-optimised prices are fully indexed to the current inflation target), the equation simplifies to the forward-looking aggregate supply curve shown in the text.

Appendix B - Optimal Policy under Discretion

Given the model in (1a), (1b) and (4), the central bank minimises its loss function each period. That is, they maximise$\left\{ \frac{1}{2}\alpha x_t^2 + (\pi_t - \pi_t^*)^2 \right\}$ with respect to x_t which yields the first order condition:

$$x_t = -\frac{\lambda}{\alpha} (\pi_t - \pi_t^*).$$

To solve for x_t and π_t in terms of shocks arriving in the model, substitute this first order condition into the Phillips curve and solve for the inflation deviation at time t:

$$\begin{aligned}
\pi_t - \pi_t^* &= \beta E_t[\pi_{t+1} - \pi_{t+1}^*] - \frac{\lambda^2}{\alpha}[\pi_t - \pi_t^*] + +u_t \\
&= \frac{\alpha\beta}{\alpha+\lambda^2} E_t[\pi_{t+1} - \pi_{t+1}^*] + \frac{\alpha}{\alpha+\lambda^2} u_t.
\end{aligned}$$

Assuming that private sector (price setters') expectations are rationally forward looking, that is $E_t[\pi_{t+1} - \pi_{t+1}^*] = E_t\left(\frac{\alpha\beta}{\alpha+\lambda^2} E_{t+1}[\pi_{t+2} - \pi_{t+2}^*] + \frac{\alpha}{\alpha+\lambda^2} u_{t+1}\right)$, repeatedly substitute and take expectations at t:

$$\begin{aligned}
\left(\alpha + \lambda^2\right)[\pi_t - \pi_t^*] &= \alpha\beta E_t\left[\frac{\alpha\beta}{\alpha+\lambda^2} E_{t+1}[\pi_{t+2} - \pi_{t+2}^*] + \frac{\alpha}{\alpha+\lambda^2} u_{t+1}\right] + \alpha u_t \\
&\vdots \\
&= \alpha E_t\left[\sum_{j=0}^{\infty}\left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^j u_{t+j}\right] \\
&= \alpha \sum_{j=0}^{\infty}\left(\frac{\alpha\beta}{\alpha+\lambda^2}\right)^j \rho^j u_t
\end{aligned}$$

Taking an infinite geometric sum yields

$$\pi_t - \pi_t^* = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} u_t.$$

and

$$x_t = -\frac{\lambda}{\alpha}[\pi_t - \pi_t^*] = \left(\frac{-\lambda}{\lambda^2 + \alpha(1-\beta\rho)}\right) u_t.$$

To find the optimal interest rate rule, employ the serial correlation in u_t to write $E_t[\pi_{t+1} - \pi_{t+1}^*] = \rho[\pi_t - \pi_t^*]$ and substitute into the policy trade-off:

$$x_t = -\frac{\lambda}{\alpha}[\pi_t - \pi_t^*] = -\frac{\lambda}{\alpha\rho} E_t[\pi_{t+1} - \pi_{t+1}^*].$$

Replacing x_t in the LHS of the aggregate demand equation and substituting for $E_t\left(x_{t+1}\right)$ in the RHS, solve for the optimal setting of the policy controlled nominal short interest rate:

$$\begin{aligned}
-\frac{\lambda}{\alpha\rho} E_t[\pi_{t+1} - \pi_{t+1}^*] &= -\gamma\left(i_t - E_t\left(\pi_{t+1}\right)\right) + E_t\left(x_{t+1}\right) + g_t \\
i_t &= \left[1 + \frac{\lambda(1-\rho)}{\alpha\gamma\rho}\right] E_t\left[\pi_{t+1} - \pi_{t+1}^*\right] + E\left(\pi_{t+1}^*\right) + \frac{g_t}{\gamma}
\end{aligned}$$

Appendix C - Steady State Kalman Filter

The following derivation of the univariate steady state Kalman gain follows the outline of Sargent's (1979) treatment of Muth's permanent income problem. From Section 3.2 we have

$$\begin{array}{lrcl} \text{Observation equation} & \pi_t & = & \pi_t^* + \delta \hat{u}_t \\ \text{State equation} & \pi_t^* & = & \pi_{t-1}^* + \varepsilon_t \end{array}$$

Add $\delta \hat{u}_t$ to both sides of the state equation (20) and rewrite the change in inflation as follows,

$$\pi_t - \pi_{t-1} = \delta(u_t - u_{t-1}) + \varepsilon_t.$$

The auto-covariance structure of this error term

$$cov\left[\delta(u_t - u_{t-1}) + \varepsilon_t, \delta(u_{t-j} - u_{t-j-1}) + \varepsilon_{t-j}\right] = \left\{ \begin{array}{c} 2\delta^2\sigma_u^2 + \sigma_\varepsilon^2 \text{ for } j = 0 \\ -\delta^2\sigma_u^2 \text{ for } j = 1 \\ 0 \text{ for } j \geqslant 2 \end{array} \right\}.$$

can be replicated by the covariance properties of the following MA process (Wold's theorem)

$$\delta(u_t - u_{t-1}) + \varepsilon_t = k_t - \phi k_{t-1} \tag{35}$$

where k_t is a stationary, serially uncorrelated random process with mean zero and variance σ_k^2 and auto-covariance properties

$$cov\left[k_t - \phi k_{t-1}, k_{t-j} - \phi k_{t-j-1}\right] = \left\{ \begin{array}{c} (1+\phi^2)\sigma_k^2 \text{ for } j = 0 \\ -\phi\sigma_k^2 \text{ for } j = 1 \\ 0 \text{ for } j \geqslant 2 \end{array} \right\}.$$

Matching coefficients there are two relationships:

$$2\delta^2\sigma_u^2 + \sigma_\varepsilon^2 = (1+\phi^2)\sigma_k^2 \text{ and} \tag{36}$$

$$-\delta^2\sigma_u^2 = -\phi^2\sigma_k^2 \tag{37}$$

which can be solved for $\sigma_k^2\left(\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}\right)$ and $\phi\left(\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}\right)$. Specifically,

$$\phi = 1 + \frac{1}{2}\left(\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}\right) - \sqrt{\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}\left(1 + \frac{1}{4}\left(\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}\right)\right)}.$$

As $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2} \to \infty$, $\phi \to 0$ and $\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2} \to 0$, $\phi \to 1$.

It will be useful to write the optimal projection as a geometrically declining lagged polynomial of previously observed inflation outcomes. From above,

$$
\begin{aligned}
(1-L)\pi_t &= (1-\phi L)k_t \\
&\Rightarrow \frac{(1-L)}{(1-\phi L)}\pi_t = \pi_t - \frac{(1-\phi)}{(1-\phi L)}\pi_{t-1} = k_t.
\end{aligned}
$$

Because k_t is orthogonal to the information set Ω_{t-1} the optimal projections are

$$
\pi_{t/t-1} = \pi^*_{t/t-1} = \pi^*_{t-1/t-1} = \frac{(1-\phi)}{1-\phi L}\pi_{t-1}.
$$

Manipulation of the above results yields

$$
\pi_t - \pi^*_{t/t} = \phi k_t \qquad \text{and}
$$

$$
\pi^*_{t/t} - \pi^*_{t-1/t-1} = (1-\phi)\left[\pi_t - \frac{(1-\phi)}{1-\phi L}\pi_{t-1}\right] = (1-\phi)k_t
$$

where $k_t = (\pi_t - \pi_{t/t-1})$ is the one-period-ahead inflation forecast error.

Appendix D - Variance of Forecast Errors

First note that the expression for the forecast error in (26) can be rewritten as

$$
i_{t+1} - i^{LI}_{t+1/t} = \varepsilon_{t+1} - \delta\hat{u}_t + \left(\pi_t - \pi^*_{t/t}\right) + \frac{\lambda}{\alpha\gamma}\delta\hat{u}_{t+1} + \frac{1}{\gamma}\hat{g}_{t+1} \tag{38}
$$

From Appendix C we have $\pi_t - \pi^*_{t/t} = \phi k_t$ so replacing this and recalling $k_t - \phi k_{t-1} = \varepsilon_t + \delta\left(u_t - u_{t-1}\right)$ we can recursively substitute for lags of k_{t-j} and group terms until the forecast error is expressed in terms of historical errors:

$$
\begin{aligned}
i_{t+1} - i^{LI}_{t+1/t} &= \varepsilon_{t+1} - \delta\hat{u}_t + \phi k_t + \frac{\lambda}{\alpha\gamma}\delta\hat{u}_{t+1} + \frac{1}{\gamma}\hat{g}_{t+1} \\
&= \varepsilon_{t+1} - \delta\hat{u}_t + \phi(\varepsilon_t + \delta\left(\hat{u}_t - \hat{u}_{t-1}\right) + \phi k_{t-1}) + \frac{\lambda}{\alpha\gamma}\delta\hat{u}_{t+1} + \frac{1}{\gamma}\hat{g}_{t+1} \\
&\vdots \\
&= \sum_{m=0}^{\infty}\phi^m\varepsilon_{t+1-m} + \frac{\lambda}{\alpha\gamma}\delta\hat{u}_{t+1} - (1-\phi)\sum_{m=0}^{\infty}\phi^m\hat{u}_{t-m} + \frac{1}{\gamma}\hat{g}_{t+1}.
\end{aligned}
$$

For $\phi < 1$ these summations are finite and the variance can be computed recalling the independence of shocks:

$$
\begin{aligned}
var(i_{t+1} - i^{LI}_{t+1/t}) &= \sigma^2_\varepsilon(1+\phi^2+\phi^4+\ldots) + \delta^2\sigma^2_{\hat{u}}\frac{\lambda}{\alpha\varphi}^2 + \delta^2\sigma^2_{\hat{u}}(1-\phi)^2(1+\phi^2+\phi^4+\ldots) + \frac{\sigma^2_{\hat{g}}}{\varphi} \\
&= \sigma^2_\varepsilon(\frac{1}{1-\phi^2}) + \delta^2\sigma^2_{\hat{u}}\left[\frac{\lambda}{\alpha\gamma}^2 + \frac{(1-\phi)^2}{1-\phi^2}\right] + \frac{\sigma^2_{\hat{g}}}{\gamma^2}
\end{aligned}
$$

Proof of Proposition 1

For any $\phi \in (0,1)$

$$\begin{aligned} \frac{1}{1-\phi^2} &> 1 \\ \frac{(1-\phi)^2}{1-\phi^2} &> 0. \end{aligned}$$

Thus for any given σ_ε^2, $\sigma_{\hat{u}}^2$ and $\sigma_{\hat{g}}^2$ and calibrated gain

$$\sigma_\varepsilon^2(\frac{1}{1-\phi^2}) + \delta^2\sigma_{\hat{u}}^2\left[\frac{\lambda}{\alpha\varphi}^2 + \frac{(1-\phi)^2}{1-\phi^2}\right] + \frac{\sigma_{\hat{g}}^2}{\varphi^2} > \sigma_\varepsilon^2 + \frac{\lambda}{\alpha\gamma}^2\delta^2\sigma_{\hat{u}}^2 + \frac{\sigma_{\hat{g}}^2}{\gamma^2}. \tag{39}$$

Appendix E - Bond Volatility

Full information: With serially correlated errors, the sum of the current and forecasted short rates is

$$i_t^{m^{FI}} = \pi_t^* + \frac{1}{m}\left(\left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right]\delta u_t\sum_{j=0}^{m-1}\rho^j + \frac{g_t}{\gamma}\sum_{j=0}^{m-1}\mu^j\right) + \zeta^m. \tag{40}$$

The one-period change in the bond is

$$i_t^{m^{FI}} - i_{t-1}^{m^{FI}} = \varepsilon_t + \frac{1}{m}\left(\left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right]\delta\sum_{j=0}^{m-1}\rho^j(u_t - u_{t-1}) + \frac{1}{\gamma}\sum_{j=0}^{m-1}\mu^j\left(g_t - g_{t-1}\right)\right) + \zeta_t^m - \zeta_{t-1}^m. \tag{41}$$

and its variance

$$var(i_t^{m^{FI}} - i_{t-1}^{m^{FI}}) = \sigma_\varepsilon^2 + \frac{1}{m^2}\left(\begin{array}{c}\left[\left[\rho + \frac{\lambda}{\alpha\gamma}(1-\rho)\right]\delta\sum_{j=0}^{m-1}\rho^j\right]^2 2(1-\rho)\sigma_u^2 \\ + \left[\frac{1}{\gamma}\sum_{j=0}^{m-1}\mu^j\right]^2 2\sigma_g^2(1-\mu)\end{array}\right) + \sigma_\varsigma^2$$

where $\sigma_u^2 = \frac{\sigma_{\hat{u}}^2}{1-\rho^2}$ and $\sigma_g^2 = \frac{\sigma_{\hat{g}}^2}{1-\mu^2}$ are the unconditional variances of the serially correlated disturbances, u_t and g_t. This variance is increasing in p, both through the first and second terms in parentheses, as well as due to the fact that $\frac{\partial\delta}{\partial\rho} > 0$. When $\rho = \mu = 0$, this expression returns the variance shown in the text for the simple case.

Limited Information: Borrowing the result from Appendix C that $\pi_{t/t}^* - \pi_{t-1/t-1}^* = (1-\phi)k_t$, it is possible to substitute recursively for k_{t-j}, $j = 0,..\infty$ using $k_t = \varepsilon_t + \delta(u_t -$

$u_{t-1}) + \phi k_{t-1}$:

$$
\begin{aligned}
i_t^{m^{LI}} - i_{t-1}^{m^{LI}} &= \frac{1}{m}\left(\varepsilon_t + (m-1)(1-\phi)k_t + \frac{\lambda}{\alpha\gamma}\delta(\hat{u}_t - \hat{u}_{t-1}) + \frac{1}{\gamma}(\hat{g}_t - \hat{g}_{t-1})\right) \\
&\vdots \\
&= \frac{1}{m}\left(\begin{array}{c} \varepsilon_t\left[1 + (m-1)(1-\phi)\right] + \frac{(m-1)(1-\phi)}{1-\phi L}\phi\varepsilon_{t-1} + \frac{1}{\gamma}(g_t - g_{t-1}) \\ + \left[\frac{\lambda}{\alpha\gamma} + (m-1)(1-\phi)\right]\delta u_t - \left[\frac{\lambda}{\alpha\gamma} + (m-1)(1-\phi)^2\right]\delta u_{t-1} \\ - \left[\frac{(m-1)(1-\phi)^2\phi}{1-\phi L}\right]\delta u_{t-2} \end{array}\right)
\end{aligned}
$$

$$
var(i_t^{m^{LI}} - i_{t-1}^{m^{LI}}) = \frac{1}{m^2}\left(\begin{array}{c} \left[\left[1 + (m-1)(1-\phi)\right]^2 + \frac{((m-1)(1-\phi)\phi)^2}{1-\phi^2}\right]\sigma_\varepsilon^2 \\ + \left[\left(\frac{\lambda}{\alpha\gamma} + (m-1)(1-\phi)\right)^2 + \left(\frac{\lambda}{\alpha\gamma} + (m-1)(1-\phi)^2\right)^2\right]\delta^2\sigma_u^2 \\ + \left(\frac{((m-1)(1-\phi)^2\phi)^2}{1-\phi^2}\right)\delta^2\sigma_u^2 + + \frac{1}{\gamma^2}2\sigma_g^2 \end{array}\right) + \sigma_\varsigma^2.
$$

Substituting the non-linear function for $\phi\left(\frac{\sigma_\varepsilon^2}{\delta^2\sigma_u^2}\right)$ it is possible to show that $var(i_t^{m^{LI}} - i_{t-1}^{m^{LI}}) > var(i_t^{m^{FI}} - i_{t-1}^{m^{FI}})$.

Appendix F - Interest Rate Regression Coefficients

For Proposition 5 to be true requires that

$$
\frac{cov(\Delta i_t^m, i_t - i_{t/t-1})^{LI}}{cov(\Delta i_t^m, i_t - i_{t/t-1})^{FI}} > \frac{var(i_t - i_{t/t-1})^{LI}}{var(i_t - i_{t/t-1})^{FI}}
$$

Forecast error variances are shown in Appendix D. The covariances are as follows:

$$
\begin{aligned}
cov(\Delta i_t^m, i_t - i_{t/t-1})^{FI} &= \sigma_\varepsilon^2 + \delta^2\sigma_{\hat{u}}^2\left[\frac{1}{m}\left(\frac{\lambda}{\alpha\gamma}\right)^2\right] + \frac{1}{m}\frac{\sigma_{\hat{g}}^2}{\gamma^2} \\
cov(\Delta i_t^m, i_t - i_{t/t-1})^{LI} &= \sigma_\varepsilon^2\left[1 + \frac{m-1}{m}\frac{\phi(1-\phi)}{1-\phi^2}\right] + \\
&\quad \delta^2\sigma_{\hat{u}}^2\left[\frac{1}{m}\left(\frac{\lambda}{\alpha\gamma}\right)^2 + \frac{\lambda}{\alpha\gamma}(1-\phi) + \frac{m-1}{m}\frac{(1-\phi)^3}{1-\phi^2}\right] + \frac{\sigma_{\hat{g}}^2}{\gamma^2}
\end{aligned}
$$

As $\frac{\sigma_\varepsilon^2}{\sigma_u^2} \to 0$, $\frac{var(i_{t+1} - i_{t+1/t}^{LI})}{var(i_{t+1} - i_{t+1/t}^{FI})} \to 1$ from above and $\frac{cov(\Delta i_{t+1}^m - i_{t+1/t})^{LI}}{cov(\Delta i_{t+1}^m, i_{t+1} - i_{t+1/t})^{FI}} \to m$ from below. As the signal-to-noise ratio rises, the covariance ratio declines monotonically toward 1. The variance ratio rises to a maximum value at ϕ' but then also declines to 1 (i.e., $\frac{\sigma_\varepsilon^2}{\sigma_u^2} \to \infty$, $\frac{var(i_{t+1} - i_{t+1/t}^{LI})}{var(i_{t+1} - i_{t+1/t}^{FI})}$ and $\frac{cov(\Delta i_{t+1}^m - i_{t+1/t})^{LI}}{cov(\Delta i_{t+1}^m, i_{t+1} - i_{t+1/t})^{FI}} \to 1$). Over the range $\frac{\sigma_\varepsilon^2}{\sigma_u^2} \in (0, \infty)$, $\frac{cov(\Delta i_{t+1}^m - i_{t+1/t})^{LI}}{cov(\Delta i_{t+1}^m, i_{t+1} - i_{t+1/t})^{FI}} > \frac{var(i_{t+1} - i_{t+1/t}^{LI})}{var(i_{t+1} - i_{t+1/t}^{FI})}$ for $m > 1$.

Appendix G - Forward rate regression coefficients

The coefficients of the following regression

$$i_{t+j/t} - i_{t+j/t-1} = \alpha + b_{2,j} \frac{\pi_t - \pi_{t/t-1}}{stdev(\pi_t - \pi_{t/t-1})} + \epsilon.$$

can be calculated as:

$$\begin{aligned}
b_{2,j}^{FI} &= \frac{cov(i_{t+j/t}^{FI} - i_{t+j/t-1}^{FI}, \pi_t - \pi_{t/t-1}^{FI})}{var(\pi_t - \pi_{t/t-1}^{FI})} \sqrt{var(\pi_t - \pi_{t/t-1}^{FI})} = \frac{\sigma_\varepsilon^2}{\sqrt{\sigma_\varepsilon^2 + \delta^2 \sigma_{\hat{u}}^2}} \quad \forall \;\; j > 1 \\
b_{2,j}^{LI} &= \frac{cov(i_{t+j/t}^{LI} - i_{t+j/t-1}^{LI}, \pi_t - \pi_{t/t-1}^{LI})}{var(\pi_t - \pi_{t/t-1}^{LI})} \sqrt{var(\pi_t - \pi_{t/t-1}^{LI})} = (1-\phi)\sigma_k \quad \forall \;\; j > 1.
\end{aligned}$$

The Kalman filter matches the change in the perceived target to the second moment of the actual target, i.e., $\sigma_\varepsilon^2 = (1-\phi)^2 \sigma_k^2$ (see matched covariances in Appendix C). Thus $b_2^{LI} = \sigma_\varepsilon$. For any $\sigma_{\hat{u}}^2 > 0$, $\sigma_\varepsilon > \frac{\sigma_\varepsilon^2}{\sqrt{\sigma_\varepsilon^2 + \delta^2 \sigma_{\hat{u}}^2}}$ and $b_{2,j}^{LI} > b_{2,j}^{FI}$.

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